INFORMATION ASYMMETRY AND STRATEGY: APPLICATIONS TO IMPARTIALITY AND FREE-RIDING

A Dissertation

by

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ABSTRACT

This dissertation consists of three essays examining information asymmetry, in particular on the topic of impartiality. The first two essays are in the form of the principle-agent model but the third essay is a public goods game where the information asymmetry is among the agents within the game.

The first essay is an implementation problem. In this juror problem, there is a true ranking of contestants unknown to the designer but known to two biased jurors. The designer's objective is to discover the true ranking. This information is unverifiable and unknowable by any means other than report from the two biased jurors. What the designer does know is the impartiality of each juror's preference. We find that to Nash implement the true ranking, the designer requires: one, every pair of contestants is an impartial pair for some juror; two, the designer knows for every contestant pair some juror for whom it is an impartial pair; three, impartial pairs are distributed such that a construction of their lower contour sets overlap.

Instead of impartiality being a restriction on preferences as in the juror problem, we next look at a literature survey of nomination rules with impartiality being a mechanism property. In this literature survey, impartiality is a property of a nomination rule that does not allow an agent to change his vote and lose his election. The literature survey establishes many impossibility results but some viable nomination rules include a majority rule with default agent and partition methods that culls an elect from partitioned (district) elections.

We finally consider a public-goods game where the charity is seeking to maximize total donations. The information asymmetry is between two types of donors, informed and uninformed. The informed donor is aware of the value of the common value public good while the uninformed donor is not. The charity has the ability to choose the cost of donors becoming informed. We find in this problem that the charity maximizes total expected donations by making information costly such that the equilibrium is a mix of informed and uninformed donors.

To my Parents who supported me always.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

A major advantage of markets is that truth-telling is an optimal strategy. The fact that truth telling is an optimal strategy means that an agent's optimal strategy need not respond to changes in the rest of the economy except by price. Thus, the great luxury of markets is ignorance. However, how ignorance distributed can create a market failure where more complex strategies are necessary to coordinate and maximize self-interest. This dissertation explores the problem of information asymmetry, its strategic consequences, and solution's trade-offs in three essays. From the the first two essays, the perspective is that of a designer of the mechanism, who, in the first essay (Section 2) is trying to design a mechanism to elicit the true ranking of contestants in a performance; and in the second essay (Section 3), the designer is attempting to create a nomination rule that does not allow the agent's vote to affect their own chances of being elected. In the third essay (Section 4), the principle is a charity choosing the cost for the agent's to learn the value of a common value public good, in order to maximize total donations.

In Section 2, we look at the problem of eliciting the true ranking of contestants. The problem is that there is a true ranking of contestants and the designer does not know this true ranking, nor can he discover it by any other means than two biased jurors. Since the jurors are biased, the designer can not simply expect them to tell him the true ranking, and since the true ranking is non-verifiable by some other means, the designer can not punish them for lying to him because he will never know which of them lied. Thus, the designer can not Nash implement the true ranking unless he knows about the juror's impartiality. A pair of contestants a and b would be an impartial pair for a juror if the designer gives the juror a choice over two rankings, where all the other contestants are in the exact same position except a and b which are adjacent and swapped, and the juror prefers the ranking where a and b are in the same order as the true ranking. Thus, from this simple choice, the designer would learn whether a is ranked before b in the true ranking, or not. The essay's results determine the distribution of impartial pairs among the two biased jurors necessary and sufficient for the designer to create a mechanism that Nash implements the true ranking.

In Section 3, we survey the literature on impartial nomination rules. In a nomination rule, each agent is both a voter and a contestant, and thus, it becomes possible for an agent to change their vote such that they lose the election when before they won the election. Rather than impartiality being used to induce truth telling by restricting preferences, this survey looks at impartiality as a property of the mechanism itself. In particular, an impartial nomination rule is a nomination rule where it is not possible for an agent to change their vote and affect their own election. The survey discusses some positive results and many impossibility results, for an example of the latter consider that there can not be anonymous ballots because an impartial nomination rule can not consider an agent's own vote in determining whether they where elected or not. The two most promising nomination rules are: one, a majority rule with a default agent, who wins when no majority found; two, a partition method nomination rule where agents are partitioned into districts, they vote, and the elect is determined from the winners of the district elections using all votes from within and without of a district.

In Section 4, we now consider a common value public goods game. As a charity, it is not designing a mechanism but choosing the cost for the agents to learn the value of the public good. The public good is a common value public good, and the information asymmetry is between the agents, who can be informed donors or uninformed donors. The strategic behavior is between the informed and uninformed donors and their attempts to balance providing the public good and free-riding. The uninformed donors begin to donate more as the expected total informed donations become increasingly pivotal to the success of the public good. As a consequence, to maximize expected total donations the charity should make it costly to learn the value of the public good such that there is a mix of informed and uninformed donors in equilibrium. Thus, the asymmetry of information among the agents in the public good's game leads to greater expected total donations, compared to if all donors had been informed or uninformed.

2. IMPLEMENTATION OF SOCIALLY OPTIMAL RANKINGS FROM TWO BIASED JURORS

2.1 Introduction

Consider the situation: You are sick. You go to a doctor, and he tells you you have a stomach ulcer, and you need to take an expensive drug. You ask him about other drugs and he talks about the generic brand, as well as some competitors, but recommends the expensive drug brand most. Often in such a situation, we hear of people getting a "second opinion" by visiting another doctor and seeing what he recommends. In this paper, we model and solve such a situation, and it turns out getting a second opinion can be worthwhile.

Past papers on a mechanism designer soliciting advice from biased advisors have generally garnered impossibility results. In Wolinsky (2002) he looks at a typical problem where they have two biased advisers and are trying to find the correct θ in the range of $[0, 1]$. He finds the designer needs advisers biased in opposite direction to get closer to the true value of θ, but finds it impossible to guarantee discovery of true θ with any observable features of the advisers. Our paper is different. It is looking for a ranking as an outcome, rather than a parameter between 0 and 1, and will use observable features of the advisers and restrictions on their preferences to get positive results. In the example of the doctor prescribing medication for ulcers, we would say a doctor is biased between recommending a drug from a company that is sponsoring him, and one that is not[1]. However, if given a choice between two rankings where only two sponsored or two non-sponsored drugs are the only drugs that change their relative ranking, then we say he is impartial between those two rankings and orders the drugs by their effectiveness. Thus, if the doctor is sponsored by both drug a and drug b, we learn about the relative effectiveness between the two drugs when he chooses between drug rankings ab or ba.

[1]He will have free samples from the sponsoring company, or you can look up the drug companies he receives gifts from in excess of $10 due to the Physician Payments Sunshine Act.

This paper is similar to Amorós (2009). He looked at implementation in a environment where he had three or more jurors. This is more common for institutional juries, such as the judges for a figure skating competition. Attempting to discover the true ranking of the figure skaters, the mechanism designer has difficulty judging the figure skaters themselves and must rely on the judges to report the true ranking of figure skaters from best performance to worst. But judges can be biased, and can seek to improve the chances of their countries candidates at the expense of others, but be impartial when ranking their own countries skaters against each other. He ultimately finds that the largest environment the designer can get the judges to reveal the true ranking in, is when he knows (1) some judge who is impartial over each contestant pair, much like the doctor above being impartial over drug a and drug b, and the designer knows this; (2) all contestant pairs have a judge impartial over them. If everyone prefers skater a to b, then we can't expect them to rank b before a, even if b out skated a.

For our results, we will require the same two assumptions. In Moore and Repullo (1990), they establish necessary and sufficient condition μ for both Nash implementation for three or more agents, and the necessary and sufficient condition $\mu 2$ for two agents. Condition μ consists of three parts that must be satisfied, the condition $\mu 2$ consists of those same three parts in condition μ, plus a part four that is unique to $\mu 2$. Thus, Amorós's results for the more general case must imply condition μ and its three parts, and our paper to extend the results to two agents must also imply the fourth part of $\mu 2$. In short, once we assume the same two assumptions as Amorós does, any additional assumptions we make are solely to imply the fourth part of condition $\mu 2$.

Given the example of two doctors choosing to report drug rankings from restricted message spaces, condition μ can be summarized as: (i) If the doctors all report the same ranking of medicine, it must be true. (ii) If a doctor reports a ranking of medicine, and that ranking is all other doctors favorite ranking, that ranking must be a true ranking. (iii) If a ranking is every doctor's favorite, it must be the true ranking. Condition $\mu 2$ adds: (iv) If the two doctors report two different rankings, the mechanism must conclude some ranking, possibly different from the two reported rankings, and that ranking must be the true ranking if both juror's prefer it to their other options. As we can see, part (iv) for condition $\mu 2$ is much more demanding than the previous three parts, because it has to cope with when the doctors disagree. When considering for three or more doctors, disagreeing reports can be ignored (by use of submechanisms that have no equilibrium). Thus, our contribution is to show two-juror implementation by satisfying part (iv) of condition $\mu 2$.

Finally, there are similar papers like Adachi (2014). He looks at ranking of contestants as an outcome as well, looking for a true ranking, but his environment has much stronger restrictions on preferences. In his own paper he admits his "impartiality except for friends" is more restrictive than the "impartial pairs " in Amorós (2009), but the reason he does this is so he can use a simple and natural mechanism to implement the true ranking. Thus, his preference restrictions are also stronger than our own.

The paper is organized as follows: Section 2 introduces the model and the environment, and formal definitions. Section 3 outlays the results and examples. Section 4 is the conclusion.

2.2 Preliminaries

2.2.1 *Basic Notation*

The model under consideration is basically the same as in Amorós (2009). There is a designer and two jurors $J = \{1, 2\}$. Let N be the set of contestants with $|N| \geq 3$. Let $n = (a, b)$ be a contestant pair, and N^2 the set of all contestant pairs[2]. A social outcome, π, is a strict ranking of the contestants in N. Denote by Π the set of all such rankings of the contestants in N, and p_a^π the position of contestant a in ranking π in which the smaller the position of a contestant is, the better his situation in the ranking would be. For example, if $\pi = (a, b, c)$, then $p_a^\pi = 1$. Accepting some sloppy notation, rankings will often be written abc, and subrankings

[2]Contestant pairs are pairs of different contestants.

as $...abc...$ when context clear. For each $n \in N^2$, we write N^2_{-n} for $N^2 \setminus \{n\}$. Let the **Set of Swapped Pairs** be a mapping $Z : \Pi \times \Pi \to 2^{N^2}$ from a pair of rankings to a set of all of the contestant pairs in different order, i.e. $Z(\pi, \pi') \equiv \{(a, b) \in N^2 | p_a^{\pi'} > p_b^{\pi'} \text{ and } p_a^{\pi} < p_b^{\pi}\}$. When the rankings π and π' are such that there is only one swapped contestant pair that is (a, b), we denote $\{(a, b)\} = Z(\pi, \pi') \cup Z(\pi', \pi)$. Note that, since it is only one contestant pair swapped, they have to be adjacent.

We assume that there is a **true ranking**, denoted by π_t, which is known by the two jurors, but unknown and unverifiable by the designer. As such, the designer wants the jurors to rank contestants truthfully. That is, the socially optimal choice rule is that the contestants should be ranked according to the true ranking.

Jurors' preferences over the set of possible rankings may depend on the true ranking. For instance, for $N = \{a, b, c\}$, a juror j may prefer ranking (b, a, c) to ranking (a, b, c) if the true ranking was $\pi_t = (b, a, c)$, but if the true ranking was $\pi_t = (a, b, c)$ then the juror prefers ranking (a, b, c) to ranking (b, a, c). The notion of preference function captures this idea. Let $\mathcal{R}$ be the set of all possible transitive preferences R_i defined over Π. Each juror j has **preference function** $R : \Pi \to \mathcal{R}$, which associates with each true ranking π_t, a preference relation $R_j(\pi_t) \in \mathcal{R}$. Thus, by the preference function, when the true ranking varies, a juror has a different preference relation. This will allow us to capture the concept of impartiality, which traditional non-state dependent preferences would not. We denote by $P_j(\pi_t)$ the strict part of juror j's preferences.

A **state of the world** is a pair $(\pi_t, R_1, R_2) \in \Pi \times \mathcal{R}^2$, where π_t is the true ranking observed by the two jurors, and (R_1, R_2) is a profile of preference relations of jurors 1 and 2. Let $E = \Pi \times \mathcal{R}^2$ be the set of all such states of the world, where $\mathcal{R}^2 = \mathcal{R} \times \mathcal{R}$. The goal of the designer is to find a mechanism that implements the socially optimal rule $f(e) = \pi_t$ for all $e = (\pi_t, R_1, R_2) \in E$.

Formally, a **mechanism** is a pair $\Gamma = (M, g)$, where $M \equiv M_1 \times M_2$ is the profile of message spaces for jurors 1 and 2, and $g : M_1 \times M_2 \to \Pi$ is an outcome function. Denote the generic message by $m \in M, m = (m_1, m_2)$.

A **Nash equilibrium** of the mechanism $\Gamma = (M, g)$ for the state of the world $e = (\pi_t, R) \in \Pi \times \mathcal{R}$, if for $m \in M$, we get $g(m)\ R_j(\pi_t)\ g(\hat{m}_j, m_{-j})$ for every $j \in J$ and $\hat{m}_j \in M_j$. Let $N(\Gamma, e)$ denote the set of Nash equilibria of Γ when the state of the world is e.

A mechanism $\Gamma = (M, g)$ is said to **Nash implement** the social optimal rule if, for all $e \in E$:

1. There exists $m \in N(\Gamma, e)$ such that $g(m) = \pi_t$.

2. If $m \in M$ is such that $g(m) \neq \pi_t$, then $m \notin N(\Gamma, \pi_t)$.

If such a mechanism exists, then the socially optimal rule is said to be *Nash implementable*.

The designer seeks to design a mechanism that Nash implements the true ranking, without the knowledge of what the preference function is. However, to have implementation, one has to restrict the domain of *possible* preference functions. A weak requirement about the impartiality of a juror over two contestants is to demand that they be an "impartial pair" for the juror. Amorós (2009) considered the class of so-called partial-impartial preferences that used knowledge of impartiality instead of bias, and proved the necessary and sufficient conditions under which the true ranking is implementable.

We will say that a juror has an **impartial pair** over (a, b), if given a choice between two rankings where a and b are adjacent, and the only difference is a and b have swapped their positions with each other, the juror strictly prefers the ranking where a and b are ordered according to the true ranking. We denote the set of juror j's impartial pairs as $I_j \subset N^2$. If some juror is impartial over all possible pairs of contestants, then the problem of electing the socially optimal ranking is trivial. The mechanism designer lets each juror choose their favorite ranking. As such, we consider a more general situation where preference functions are partially-impartial. Formally, we have:

Definition 1 (Partial-Impartialness of Preferences)**.** Given the set of impartial pairs for juror j, I_j, the preference function $R_j : \Pi \to \mathcal{R}$ is **partially-impartial** for j if whenever for $j \in J$,

(1) $I_j \subsetneq N^2$ (i.e., not all pairs are impartial), and

(2) for each $(a, b) \in I_j$, each $\pi_t \in \Pi$, and each pair $\{\pi, \hat{\pi}\} \subset \Pi$ with:

(i) $p_a^{\pi} = p_b^{\pi} - 1$;

(ii) $p_a^{\hat{\pi}} = p_b^{\hat{\pi}} + 1$;

(iii) $p_c^{\pi} = p_c^{\hat{\pi}}$ for all $c \in N \setminus \{a, b\}$;

(iv) $p_a^{\pi_t} < p_b^{\pi_t}$,

we have $\pi P_j(\pi_t)\hat{\pi}$.

Denote by $\hat{I}_i \subseteq I_i$ the set of impartial pairs of juror i known by the designer. The following example clarifies the notion of partial-impartial preferences so that the preference functions $R_j : \Pi \to \mathcal{R}$ is partially-impartial for j.

Example 1. Suppose juror j is ranking three gymnastics contestants **A**lice, **B**arbara, and **C**hristina in a gymnastics competition. As such $N = \{a, b, c\}$ and $N^2 = \{(a, b), (a, c), (b, c)\}$. Consider the case where the designer knows the juror is impartial over Alice and Christina, because they are her friends, $(a, c) \in \hat{I}_j$, but she is biased between Barbara and Alice because only Alice is a friend, $(a, b) \notin \hat{I}_j$. Then, if the juror was given a choice over the rankings (Barbara, Alice, Christina) and (Barbara, Christina, Alice), hereafter written bac and bca, she would choose the ranking where Alice and Christina are in the same order as the true ranking.

Suppose the true ranking is $\pi_t = abc$, then the juror would prefer ranking $\pi = bac$ to ranking $\hat{\pi} = bca$. Then, it is clear that $p_a^{\pi} = p_c^{\pi} - 1$, $p_a^{\hat{\pi}} = p_c^{\hat{\pi}} + 1$, $p_b^{\pi} = p_b^{\hat{\pi}}$, and $p_a^{\pi_t} < p_c^{\pi_t}$. Further, since $(a, b) \notin \hat{I}_j$, the preference function $R_j : \Pi \to \mathcal{R}$ is partially-impartial for j. Yet, even though the juror is biased, we could get some meaningful information from them by just knowing Alice and Christina are her friends.

From $(a, c) \in \hat{I}_j$ and $C_j(bac, bca) = bac$, then the designer can infer that π_t is either abc or bac, but not bca.

As Amorós (2009) pointed out, because of $I_j \subsetneq N^2$, there always is a $\pi_t \in \Pi$ and $\pi, \hat{\pi} \in \Pi$ where only two consecutive contestants, who are not an impartial pair, change their relative positions, and that, when comparing π with $\hat{\pi}$, juror j does not strictly prefer that ranking where these two contestants are truthfully placed. This is the reason why we say that juror j is partially-impartial, which allows for the possibility that the jurors be biased in many different ways. For example, a juror may have "friends" and/or "enemies" among the contestants.

2.2.2 *Existing Results*

For the case of $|J| \geq 3$, Amorós (2009) provided the necessary and sufficient conditions on jurors under which the socially optimal rule is Nash implementable. Amorós (2009) first showed the following conditions are necessary for Nash implementation of the socially optimal rule for any number of jurors. As such, we also assume these conditions in the current paper and state them here.

Proposition 1 (Amorós (2009))**.** *Suppose that preference functions are partially-impartial. If the socially optimal rule is Nash implementable, then every pair of contestants must be an impartial pair for at least one juror, and the designer of the mechanism must know, for every pair of contestants, at least one of the jurors who have them in their impartial pairs.*

The proofs of the above propositions do not depend on Nash being the equilibrium concept, and these conditions are necessary for implementation of the socially optimal rule in any traditional solution concept of equilibrium. If there is a pair of contestants that is not an impartial pair for any juror, or is an impartial pair of an unknown juror, then there are some states of the world in which the preferences of the jurors are the same although the true ranking is different. As such, there is no way of implementing the socially optimal rule if the designer of the mechanism has any uncertainty concerning the juror for whom some pair of contestants is an impartial pair.

From now on we assume preferences are partially-impartial, (1) that every pair of contestants is an impartial pair for at least one juror and (2) that, for each pair of contestants, the designer of the mechanism knows at least one juror having them among their impartial pairs.

However, unlike what happens in most economic environments, the socially optimal (true ranking) rule does not satisfy no-veto power.[3] But, that does not imply that the socially optimal rule fails to be Nash implementable, since no-veto power is not a necessary condition, although it is sufficient for Nash implementation when $|J| \geq 3$. Indeed, for the case of $|J| \geq 3$, Amorós (2009) further proved that these conditions are also sufficient for Nash implementation of the socially optimal rule. That is, the socially optimal rule is Nash implementable under the necessary conditions specified in the above propositions as stated in the following theorem.

Theorem 1 (Amorós (2009))**.** *Suppose that preference functions are partially-impartial. If (i) there are at least three jurors, (ii) every pair of contestants is an impartial pair for at least one juror and (iii) the designer of the mechanism must know, for every pair of contestants, at least one of the jurors who have them in their impartial pairs. Then the socially optimal rule is Nash implementable.*

Amorós (2009) shows in Appendix A, that these conditions are not sufficient for Nash Implementation when there are only two jurors. To have a sufficiency result, one has to impose additional conditions. Then an open question is that under what additional conditions, the socially optimal rule is Nash implementable in the case of the two jurors. This paper fills the gap.

2.3 New Results

In Moore and Repullo (1990), for Nash Implementation in the case of only two jurors, they introduce the following condition, called Condition $\mu 2$. We state this condition in the setting of this paper.

[3] A social choice rule $f : E \to A$ is said to satisfy no-veto power if whenever for any i and e such that $x \succcurlyeq_j y$ for all $y \in A$ and all $j \neq i$, then $x = f(e)$.

Definition 2 (Condition $\mu 2$)**.** The socially optimal choice rule f is said to satisfy condition $\mu 2$ if there is a set $B \subseteq \Pi$ and, for each $i \in J$ and a state of the world $(\pi_t^*, R) \in \Pi \times \mathcal{R}^2$, there exists a set $C_i(\pi_t^*) \subseteq B$, with $\pi_t^* \in M_i(C_i(\pi_t^*), \pi_t^*)$ such that for all $\pi_t, \hat{\pi}_t \in \Pi$, the following are satisfied:

(i) if $\pi_t \in M_i(C_i(\pi_t), \pi_t^*) \cap M_j(C_j(\pi_t), \pi_t^*)$, then $\pi_t = \pi_t^*$;

(ii) if $\pi \in M_i(C_i(\pi_t), \pi_t^*) \cap\ M_j(B, \pi_t^*)$ for $i \neq j$, then $\pi = \pi_t^*$;

(iii) if $\pi \in M_i(B, \pi_t^*) \cap M_j(B, \pi_t^*)$, then $\pi = \pi_t^*$;

(iv) there exists $\pi \in C_i(\pi_t) \cap C_j(\hat{\pi}_t)$ and if $\pi \in M_i(C_i(\pi_t), \pi_t^*) \cap M_j(C_j(\hat{\pi}_t), \pi_t^*)$, then $\pi = \pi_t^*$.

Notice in Moore and Repullo (1990), B is a set of all the outcomes of a mechanism that nash implements the true ranking. Since any ranking can be the true ranking, that means $B = \Pi$. Further, $C_i(\pi)$ denotes the *range of outcomes* that juror i can generate by varying his own strategy, keeping the other juror's strategy constant. For implementation, it is necessary for the *range of outcomes* of juror i to be a *full range of outcomes* for both juror 1 and 2. The **full range of outcomes** l_i and l_j, satisfies three criteria for any two rankings: first, it must be a subset of the known lower contour set of π, for any ranking, using what is known of juror i's impartial pairs; second, the juror's range of outcomes must overlap; and third, for any contestant pair (a, b), there must be either l_i or l_j that have two rankings which are identical except (a, b) are adjacent and swapped. Notice that for a C_1 that satisfies only the first two requirements (achieved by a proper subset of some full range of outcomes l_1 such that $C_1 \subsetneq l_1$), would be only necessary, not sufficient.

Definition 3 (Full Ranges of Outcomes)**.** *Define the* **full ranges of outcomes** l_i *and* l_j *as two range of outcomes* C_i *and* C_j *such that, for any two rankings* $\pi_i, \pi_j \in \Pi$*,* C_i *and* C_j *satisfies:*

1. $C_i(\pi_i) \subseteq L_i(\pi_i) \equiv \{\hat{\pi} \in \Pi | Z(\pi_i, \hat{\pi}) \cup Z(\hat{\pi}, \pi_i) \subseteq \hat{I}_i\}$*, for both jurors* i, j*.*

2. *$C_i(\pi_i) \cap C_j(\pi_j)$ is nonempty.*

3. *For any $(a,b) \in N^2$, there exists rankings $\hat{\pi}, \tilde{\pi} \in \Pi$ such that $\{(a,b)\} = Z(\hat{\pi}, \tilde{\pi})$, and $\{\hat{\pi}, \tilde{\pi}\} \subseteq C_i(\pi_i)$ or $\{\hat{\pi}, \tilde{\pi}\} \subseteq C_j(\pi_j)$.*

then C_i and C_j are a full range of outcomes, denoted by $C_i = l_i$ and $C_j = l_j$.

Given the assumptions from Amorós (2009), that preference functions are partially-impartial[4], (1) every pair of contestants is an impartial pair for at least one juror, (2) the designer of the mechanism must know, for every pair of contestants, at least one of the jurors who have them in their impartial pairs, we are able to deduce a lemma that will prove integral to showing necessary and sufficient conditions for Nash implementation. The lemma identifies for any non-true ranking, an adjacent impartial pair that is swapped from the true ranking, and identifies the juror i who is known to be impartial over that pair.

Lemma 1. *For two-juror economic environments under consideration. Suppose that preference functions are partially-impartial, (1) every pair of contestants is an impartial pair for at least one juror, (2) the designer of the mechanism must know, for every pair of contestants, at least one of the jurors who has them in their impartial pairs. Then for any ranking $\pi \neq \pi_t$, the designer knows there exists a swapped pair (a,b) that is adjacent in π, and juror i for whom it is an impartial pair.*

Thus, with the lemma, we know that if the designer is given a ranking $\pi \neq \pi_t$, he knows: First, a swapped pair (a,b) that is adjacent in π, and second, a juror i for whom the swapped pair (a,b) is an impartial pair, $(a,b) \in \hat{I}_i$. With (a,b) known, we will be able to identify another ranking π' that is similar to π, but known to be strictly preferred by juror i. This will be useful to disprove false equilibria in the proof of the theorem.

[4]He explicitly assume partially-impartial preferences since, with fully impartial preferences, we do not need more than one juror. Thus, to look for necessary characterization that requires two jurors, we will also need to assume partially-impartial preferences to ensure the second juror is not redundant.

With the three assumptions from Amorós (2009) we can to prove parts (i), (ii), and (iii), but to satisfy part (iv), we will need an additional assumption: That the sets C_i and C_j always have a non-empty intersection, which thus includes sets l_1 and l_2 too. Formally:

Definition 4 (Non-empty Intersection)**.** *There exists* $\pi \in \Pi$*, such that* $\pi \in C_1(\pi') \cap C_2(\pi'')$ *for any* $\pi', \pi'' \in \Pi$.

The assumption of the Non-empty Intersection and Amorós three assumptions will be both necessary and sufficient. From Amorós (2009), Appendix A, we already know that his conditions are necessary for implementing with two jurors, but not sufficient. The following example shows that the three assumptions from Amorós (2009) do not imply the assumption of Non-empty Intersection.

Example 2. *Let the true ranking be* $\pi_t = bac$*, and let the impartial pairs for juror* 1 *be* $\hat{I}_1 = \{(a,b),(a,c)\}$*, and the impartial pairs for juror* 2 *be* $\hat{I}_2 = \{(b,c)\}$*. Let* $\pi = bac$ *and* $\pi' = cab$*.*

Then $C_1(bac) = \{abc, bac, bca\}$ *and* $C_2(cab) = \{cab\}$ *and notice they have no rankings in common.*

As a result, even when satisfying the the three assumptions of preference functions are partially-impartial, (1) every pair of contestants is an impartial pair for at least one juror, (2) the designer of the mechanism must know, for every pair of contestants, at least one of the jurors who has them in their impartial pairs, we are able to construct a counter example that shows for the rankings $\pi = bac$ *and* $\pi' = cab$*, that the sets* C_i *and* C_j *have an empty intersection. Therefore, assuming (3) is not redundant.*

Thus, the assumption of non-empty intersection is not redundant. With the three assumptions from Amorós (2009) and Non-empty Intersection, it is possible to implement the true ranking, and if you implement the true ranking, you also satisfy those four assumptions. Thus, they are necessary and sufficient if we seek to design a jury that allows us to elicit the true ranking.

Theorem 2. *For two-juror economic environments with preference functions that are partially-impartial. The socially optimal rule that is given by the true ranking of contestants is Nash implementable if and only if, (1) every pair of contestants is an impartial pair for at least one juror, (2) the designer of the mechanism must know, for every pair of contestants, at least one of the jurors who have them in their impartial pairs, and (3) there exists* $\pi \in \Pi$, *such that* $\pi \in C_1(\pi') \cap C_2(\pi'')$ *for any* $\pi', \pi'' \in \Pi$.

Proof. Set $B = \Pi$, because it is the range of mechanism that Nash implements the true ranking which can be any ranking, and take $C_i(\pi_t) = l_i(\pi_t)$. The proof proceeds by first showing a claim, then showing necessity, then showing sufficiency by sequentially proving the theorem's assumptions imply part (i), part (ii), part (iii), and then part (iv) of $\mu 2$ from Amorós (2009).

Claim 1: $\hat{I}_i$ *is nonempty when juror* i *satisfies partially-impartial preferences,(1) every pair of contestants is an impartial pair for at least one juror, (2) the designer of the mechanism must know, for every pair of contestants, at least one of the jurors who have them in their impartial pairs.*

Suppose preferences partially-impartial, (1), and (2) from above. Proof of claim proceeds by way of contradiction, so assume that $\hat{I}_j$ is empty.

By partially-impartial preferences we know that for agent i, there exists a contestant pair $(a, b) \notin I_i$. Thus, we know that $(a, b) \in I_j$, for $j \neq i$, since the designer knows that every contestant pair is an impartial pair for at least one juror. Since $\hat{I}_i \subseteq I_i$, we also know that $(a, b) \notin \hat{I}_i$.

Then, since the designer knows that, for every pair of contestants, a juror who has them as an impartial pair, he must know that either juror i or juror j has (a, b) as an impartial pair. By $(a, b) \in I_j$ we know juror j *could* have $(a, b) \in \hat{I}_j$, and since $(a, b) \notin \hat{I}_i$, by process of elimination, it has to be juror j. Therefore, $\hat{I}_j$ is not empty. A contradiction, and thus shows that with two jurors, and preferences partially-impartial, (1), and (2) that $\hat{I}_i$ and $\hat{I}_j$ is never empty. This proves the claim. ✓

Show Necessity $(\rightarrow)$

Suppose that C_i and C_j Nash implement the true ranking. From Amorós (2009), we know the first three assumptions are necessary. What remains to be shown is necessity of Non-empty Intersection, and that $C_1 = l_1$ and $C_2 = l_2$.

Suppose that the true ranking is Nash implementable. Since the true ranking is Nash implementable, that means it satisfies Moore and Repullo (1990) condition $\mu 2$. Look at part (iv) of $\mu 2$:

> (iv) there exists $\pi \in C_i(\pi_t) \cap C_j(\hat{\pi}_t)$ and if $\pi \in M_i(C_i(\pi_t), \pi_t^*) \cap M_j(C_j(\hat{\pi}_t), \pi_t^*)$,
> then $\pi = \pi_t^*$.

Notice, the first half of part (iv) of $\mu 2$ is exactly the assumption of Non-empty Intersection. Therefore, $\mu 2$ implies Non-empty Intersection.

Now to see that C_1 and C_2 must satisfy the three conditions to be l_1 and l_2:

1) satisfied because C_1 and C_2 are a subset of the knowable lower contour set L_1 and L_2 too.

2) satisfied by condition of $\mu 2$ part (iv).

3) will be shown by way of contradiction: For two rankings π, π', there exists some contestant pair $(a, b) \in N^2$, such that there does not exist rankings $\hat{\pi}, \tilde{\pi} \in \Pi$ in either $C_i(\pi)$ or $C_j(\pi')$ where (a, b) is adjacent and swapped.

Now, let $\hat{\pi} \in C_1(\pi) \cap C_2(\pi')$ and let π_t be such that $Z(\hat{\pi}, \pi_t) \cup Z(\pi_t, \hat{\pi}) = \{(a, b)\}$ and let (a, b) be an impartial pair of juror 1.

Since the ranking $\hat{\pi} \in C_1(\pi) \cap C_2(\pi')$, and the true ranking is only differentiated by its adjacent contestants a and b being swapped, by assumption of contradiction, we know that the true ranking is in neither range of outcome, i.e. $\pi_t \notin C_1(\pi) \cup C_2(\pi')$. Therefore, what remains to be shown is that $\hat{\pi}$ is the most preferred ranking of the choices for both jurors 1 and 2.

To see that $\hat{\pi} \in M_1(C_1(\pi), \pi_t)$, notice the three following: one, $\hat{\pi}$ has all contestant pairs ranked as the true ranking, except (a, b); two, the true ranking is not an option; three, all rankings in C_1 are different from one another by swapping adjacent impartial pairs of juror 1 alone, which means any other ranking in C_1 that is not $\hat{\pi}$ must be swapping impartial pairs that are already in the correct order, which by partially-impartial preferences immediately implies $\hat{\pi}$ is preferred to them. Therefore, $\hat{\pi} \in M_1(C_1(\pi), \pi_t)$. By symmetry, the same argument shows $\hat{\pi} \in M_2(C_2(\pi'), \pi_t)$ for juror 2. Therefore, $\hat{\pi} \in M_1(C_1(\pi), \pi_t) \cap M_2(C_2(\pi'), \pi_t)$, and thus $\hat{\pi}$ is an equilibrium and is not the true ranking, a contradiction of C_1 and C_2 implementing the true ranking. Therefore, if the mechanism Nash implements the π_t, then C_1 and C_2 must satisfy all three conditions to be a full range of outcome l_1 and l_2, and Non-empty Intersection, preference's partially-impartial and assumptions (1) and (2). Thus, necessity is shown.

Show Sufficiency $(\leftarrow)$

We proceed to prove that with preferences partially-impartial and assumptions (1), (2), and (3) imply parts (i),(ii), (iii), and (iv) of $\mu 2$. We do so one at a time.

Proof of (i): Let $\pi_t \in M_i(l_i(\pi_t), \pi_t^*) \cap M_j(l_j(\pi_t), \pi_t^*)$. We want to show that $\pi_t = \pi_t^*$. Suppose, by way of contradiction, that $\pi_t \neq \pi_t^*$. Note that for contestant pairs in $Z(\pi_t^*, \pi_t) \cup Z(\pi_t, \pi_t^*)$, there must be a contestant pair (a, b) that is adjacent, by lemma 1. Without loss of generality, let $(a, b) \in \hat{I}_1$. Let the ranking π be such that $Z(\pi_t, \pi)$ contains only (a, b) (in which case they have to be adjacent under π_t and π). As such, $\pi \in l_1(\pi_t)$, and thus, by Partial-Impartial Preferences, we know that $\pi P_1(\pi_t^*)\pi_t$, which contradicts the fact that $\pi_t \in M_1(l_1(\pi_t), \pi_t^*)$. Therefore $\pi_t = \pi_t^*$. Thus (i) is shown.

Proof of (ii): Suppose that by way of contradiction that there exists a ranking π such that $\pi \in M_i(l_i(\pi_t), \pi_t^*) \cap M_j(\Pi, \pi_t^*)$ for $i \neq j$ and $\pi \neq \pi_t^*$. By lemma 1, there must be a swapped pair (a, b) going from π to the true ranking π_t^*, that is adjacent in π. That gives us two cases: Case 1, $(a, b) \in \hat{I}_i$, or case 2, $(a, b) \in \hat{I}_j$.

If (a, b) is in juror i's impartial pairs, then repeat the proof from part (i) to come to a contradiction. Thus, consider the second case, where $(a, b) \in \hat{I}_j$. Like in part (i)'s proof, choose a ranking π' such that $Z(\pi, \pi') = \{(a, b)\}$, where the only difference between the rankings π

and π' is that the adjacent contestant pair (a, b) is swapped. Since this was swapped pair between π and the true ranking π_t^*, that means by definition of partially-impartial preferences that $\pi' P_j(\pi_t^*)\pi$, a contradiction of π be juror j's most preferred ranking of all rankings. Thus we exhaust both cases, and show (ii).

Proof of (iii): Proof same as shown in (ii) for case 2. Assume by way of contradiction that π is most preferred ranking for all jurors, and it isn't the true ranking. By lemma 1, there is swapped pair (a, b), where $(a, b) \in \hat{I}_i$ for some juror i, and contestants a and b are adjacent in π. Then, choose ranking π' that is the same as ranking π except the adjacent a and b are swapped, $\{(a, b)\} = Z(\pi, \pi') \cup Z(\pi', \pi)$. Then, by definition of partially-impartial preferences, we know that π' is strictly preferred to π, thus contradicting π being juror i's most preferred ranking. This shows (iii).

Proof of (iv): By Condition (3), we know there is a ranking $\pi \in l_1(\pi_t) \cap l_2(\hat{\pi}_t)$. By way of contradiction, let $\pi \neq \pi_t^*$ and $\pi \in M_1(l_1(\pi_t), \pi_t^*) \cap M_2(l_2(\hat{\pi}_t), \pi_t^*)$.

Given $\pi \neq \pi_t^*$, we apply lemma 1 on $l_i(\pi_t)$. If, for π_t, there is a swapped adjacent pair $(a, b) \in \hat{I}_i$, great, proceed to next paragraph; otherwise, if all swapped adjacent pairs for π_t are in juror $j's$ impartial pairs, then continue this paragraph. Since $\pi \in l_i(\pi_t)$, we know that there is an adjacent impartial pair for juror j for π, and because $\pi \in l_j(\hat{\pi})$, we know there is a ranking in $l_j(\hat{\pi})$ that has an adjacent swapped pair in juror $j's$ impartial pairs. By definition of l_j, we know that $Z(\pi, \hat{\pi}) \cup Z(\hat{\pi}, \pi) \subseteq \hat{I}_j$, implying that there is adjacent impartial pairs of juror j for $\hat{\pi}$. Thus, there exists swapped adjacent $(a, b) \in \hat{I}_j$ for $\hat{\pi}$, and it is inconsequential whether the adjacent impartial is for juror i or juror j.

Without loss of generality, let contestant pair (a, b) in $Z(\pi, \pi_t^*) \cup Z(\pi_t^*, \pi)$ be adjacent for π and let the designer know (a, b) is juror 1's impartial pair. Choose π' such that $\{(a, b)\} = Z(\pi', \pi)$. As a result, $\pi' \in l_1(\pi_t)$, and by definition of partially-impartial preferences $\pi' P_1(\pi_t^*)\pi$, a contradiction of $\pi \in M_1(l_1(\pi_t), \pi_t^*)$, therefore it must be that $\pi = \pi_t^*$.

Hence, Moore and Repullo's condition $\mu 2$ is satisfied and thus the socially optimal rule is

Nash implementable. $\square$

Thus, by adding the non-empty intersection, we update Amorós's results to be *both* necessary and sufficient when there are only two jurors. For illustration, we show an example below.

2.4 Example

Define $C_i(\pi') \equiv \{\pi \in \Pi | Z(\pi, \pi') \cup Z(\pi', \pi) \subseteq \hat{I}_i\}$. Let the known impartial pairs for the two jurors be: $\hat{I}_1 = \{(a, b), (a, c)\}$, and $\hat{I}_2 = \{(a, b), (b, c)\}$. So far, this satisfies Amorós (2009) three assumptions, that all contestant pairs be an impartial pair for some juror, that the designer knows for each contestant pair a juror who has it as an impartial pair, and the preferences are partially-impartial. What remains to be shown is that the sets C_1 and C_2 overlap.

$$
\begin{aligned}
C_1(abc) = C_1(bac) = C_1(bca) &= \{abc, bac, bca\} \\
C_1(acb) = C_1(cab) = C_1(cba) &= \{acb, cab, cba\}
\end{aligned}
$$

$$
\begin{aligned}
C_2(bac) = C_2(abc) = C_2(acb) &= \{bac, abc, acb\} \\
C_2(bca) = C_2(cba) = C_2(cab) &= \{bca, cba, cab\}
\end{aligned}
$$

Thus, the following intersections are: $C_1(abc) \cap C_2(bac) = \{abc, bac\}$, $C_1(abc) \cap C_2(bca) = \{bca\}$, $C_1(acb) \cap C_2(bac) = \{acb\}$, and $C_1(acb) \cap C_2(bac) = \{cba, cab\}$. Thus, this simple example also satisfies *Non-empty Intersection*.

Consider a simple mechanism, where the jurors only report the position of the juror for whom they have all of the impartial pairs that contains that juror. In this example, juror 1 would report contestant a's position as either first, second, or third, and juror two would report contestant b's position as either first, second, or third. The outcome function g would then take juror 1 and 2's report and find the ranking that satisfies their reports. If juror 1 reports contestant

a was third, and juror 2 reports contestant b was second, then the outcome $g(3,2) = cba$. If both juror 1 and 2 report the same position, then the output function gives the position to contestant a, and positions b in the available adjacent position, then places c in the last available position.

Formally, the message space: $m_1 \in M_1 \equiv \{1,2,3\}$ and $m_2 \in M_2 \equiv \{1,2,3\}$.

The outcome function that reports a ranking from juror 1 and 2's report of contestant positions is straightforward when they report two different positions (case 1), but when they report the same position (case 2), then the reported position is given to contestant a as a tie breaker. Formally:

$$g(m_1, m_2) = \{\pi \in \Pi | \begin{cases} p_a^\pi = m_1, p_b^\pi = m_2, \text{ and fit } c, & \text{if } m_1 \neq m_2. \\ p_a^\pi = m_1, \begin{cases} p_b^\pi = m_2 + 1, & \text{if } m_2 \leq 2. \\ p_b^\pi = m_2 - 1, & \text{otherwise.} \end{cases} \text{ and fit } c, & \text{otherwise.} \end{cases}$$

Thus, for example, if juror 1's message was $m_1 = 1$, and juror 2's message was $m_2 = 2$, then $g(1,2) = abc$[5].

Now let the true ranking be $\pi_t = abc$. Below, we show how all possible rankings that are not the true ranking are not an equilibrium.

$g(3,2) = cba$ but Juror 1 prefers $g(2,2) = cab$ because $(a,b) \in \hat{I}_1$.
$g(3,1) = bca$ but Juror 1 prefers $g(2,1) = bac$ because $(a,c) \in \hat{I}_1$.
$g(2,3) = cab$ but Juror 1 prefers $g(1,3) = acb$ because $(a,c) \in \hat{I}_1$.
$g(2,1) = bac$ but Juror 1 prefers $g(1,1) = abc$ because $(a,b) \in \hat{I}_1$.
$g(1,3) = acb$ but Juror 2 prefers $g(1,2) = abc$ because $(b,c) \in \hat{I}_2$.
$g(1,2) = abc$ and is the true ranking.

So no other ranking than the true ranking is an equilibrium. What remains to be shown, is

[5] Additional examples: $g(3,2) = cba$, $g(1,1) = abc$, and $g(3,3) = cba$.

that the true ranking is an equilibrium. For juror 1, all of his choices for $g(1,2)$ would include $g(2,2), g(3,2)$, which gives $\{abc, bac, bca\} = C_1(abc)$, which is the lower contour set of abc, therefore the true ranking is most preferred. Likewise, the options for juror 2 are $g(1,1) = abc, g(1,2) = abc$, and $g(1,3) = acb$. And we know abc preferred to bac from the above. Similarly, if $g(1,1) = abc$, then both jurors' have the same choice of outcomes, and therefore prefer the abc. Thus, the true ranking is an equilibrium.

This simple mechanism implements the true ranking with two jurors when its satisfies all four assumptions. Also, the example isn't unique. If the known impartial pairs were $\hat{I}_1 = \{(a,c),(b,c)\}$, it would still work. Juror 1 would report contestant c's position instead of contestant a's. So long as both jurors had known impartial pairs that had all of the impartial pairs for some contestant, we can get implementation.

2.5 Conclusion

Where Amorós (2009) shows implementation for three or more jurors, we extend his results to when you have only two jurors. Thus, if a patient had biased doctors advising him, and needed to extract the true ranking of medicine for himself, he would be able to determine if the reports from just two doctors was enough, and whether he needed to go see a third doctor. In real life, such a cost to finding more advisers can be great, and knowing when you don't need to can save money. Solving for the two juror case makes that decision possible.

To get Nash Implementation of the true ranking with two juror's, however, we required an additional assumption: that the impartiality is such ordered that the two jurors's lower contour sets deduced from known impartiality always overlap with each other. Combining this additional assumption with the three assumptions from Amorós paper identifies the necessary and sufficient characterization of two juror Nash Implementation. If Amorós results are interpreted as a negative result, mine, which are more restrictive, must also be considered a negative result.

3. IMPARTIALITY IN DESIGN OF NOMINATION RULES

3.1 Introduction

Economics evolved from Political Economy, and as far back as the 1700s with the Marquis de Condorcet, we see methodical research into voting mechanisms. A defining feature of the voting literature has been impossibility results; the canonical result of the Gibbard-Satterhwaite Theorem being from the 1970s, respectively Gibbard (1973) and Satterthwaite (1975). To underscore its importance, new proofs for the theorem continue to be published, while changes in its assumptions are tested to get positive results. Often, by restricting preferences or weakening strategy-proofness. This paper surveys literature that weakens the strategy-proof restriction to impartiality.

Theorem 3 (Gibbard-Satterthwaite Theorem)**.** *Suppose there are at least three alternatives and that for each individual any strict ranking of these alternatives is possible. Then a social choice function is strategy-proof only if it is a dictatorship.*

In this Literature Survey, we will consider nomination rules where the voters and contestants are the same. We will assume at least three agents, and that each agent's preferences are unrestricted. We will weaken strategy-proof requirement to impartiality, and allow agents's votes and candidacy to be treated differently. An agent's candidacy is treated differently, if, they needed more votes or votes from particular voters, unlike some other agent, to be elected. Thus, we will be looking at the tradeoff between a nomination rule satisfying optimality of simple strategies, voter equity, and candidate equity.

Because the voters and the contestants are the same, we introduce the possibility that a person votes for himself, even though they think someone else is better qualified for the job. When an agent's vote does not affect his or her own candidacy, we say that mechanism satisfies impartiality. An impartial mechanism is a weaker requirement than being a strategy-proof mechanism, thus optimal strategies may be more complex than truth telling.

If we have a dictatorial nomination rule, that means we treat some agent's vote with infinitely more weight than the other agents' votes. The opposite of this is when everyone's vote is the exact same weight, which we call anonymous ballots. This is a strong condition; for example, the US presidential election does not satisfy this property because rural voters have greater relative strength to urban votes in the Electoral College. A weaker condition for voter equity is no-dummy: anyone's vote could affect the outcome.

Further, a possible solution to the voting impossibility theorems is a simple majority rule with only two contestants. In our environment, the candidates are the voters too, and we assume more than two voters. We will assume a property called no-exclusion, which ensures anyone is a potential nominee. A common finding is that in order to satisfy impartiality and voter equity, we create inequity among candidates.

We will finally consider negative unanimity. Negative unanimity is when an agent no one voted for cannot become the nominee. Surprisingly, it is difficult for a mechanism to satisfy impartiality and negative unanimity.

The paper will introduce the Model in section 2, and in section 3, the results for impartial nomination rules, with subsections: impartiality alone, anonymous ballots, Restricted Message Space and candidacy equity, and negative unanimity. First, we discuss the model.

3.2 Model

Given a finite set of agents N, with $|N| = n > 3$. Let the vote profiles be denoted $N_-^N \equiv \{x \in N^N | x_i \in N \setminus \{i\}, \text{ for each } i\}$.

Definition 5. *A nomination rule is a function $\varphi : N_-^N \to N$.*

Given a profile of votes $x \in N_-^N$, the score of votes for contestant i is given by $s_i = |\{j \in N | x_j = i\}$, and denote the profile of scores as $\delta(x) = s$. Denote the vote of all other contestants than i as x_{-i}, and the set of all such profiles is $N_{-}^{N\setminus\{i\}}$. Given the notation of $x_{-i} \in N_{-}^{N\setminus\{i\}}$, and i's vote $x_i \in N \setminus \{i\}$, and the profile as $(x_i, x_{-i}) \in N_{-}^{N}$, we can now define the impartiality.

Definition 6 (Impartiality)**.** *For all* $i \in N, x_i, x_i' \in N \setminus \{i\}$*, and all* $x_{-i} \in N_{-}^{N\setminus i}$*,*

$$\varphi(x_i, x_{-i}) = i \iff \varphi(x_i', x_{-i}) = i.$$

Thus, a nomination rule is impartial, if and only if, when under the vote profile x agent i is nominated, then i must be nominated under any other vote profile (x_i', x_{-i}) where only agent i's vote x_i is different. Thus, agent i cannot change his vote to *anything* to suddenly not be nominated. This does not prevent agent i from changing his vote to affect other people's chances of winning when he is not nominated, and in those circumstances, he can lie. Thus, *impartiality* is weaker than requiring truth telling.

Definition 7 (Anonymous Ballots)**.** *For all* $x, y \in N_{-}^{N}$*,*

$$\delta(x) = \delta(y) \implies \varphi(x) = \varphi(y).$$

Notice that with anonymous ballots, the nominee is determined by the vote count alone. Specifically, an unweighted sum of votes such that all votes are equal. For example, in the United States Electoral College, the winner of the United States presidential election is determined by votes from each state. Each state has a flat two votes, plus additional votes for population. Because of this, the votes from voters in low population states are weighted more than the individual votes of voters in high population states. Thus, the United States presidential election is a jurisdiction-weighted vote count. This violates anonymous ballots. However, in this same election, your vote is "anonymous" in the sense it is untraceable to protect the voter from coercion, which is called a secret ballot.

Next are additional properties we seek in our nomination rules:

No-Dummy: No adviser has their nomination always ignored: for all $i \in N$, there exists $x_i, x_i' \in N \setminus \{i\}$ and $x_{-i} \in N_-^{N\setminus i}$ such that:

$$\varphi(x_i, x_{-i}) \neq \varphi(x_i', x_{-i}).$$

No-Exclusion: There always exists some profile of nominations such that any adviser can be the one nominated by the nomination rule: for all $i \in N$, there exists $x \in N_-^N$ such that $\varphi(x) = i$.

Negative Unanimity: If no one nominated an adviser, that adviser cannot be nominated by the rule: for all $x \in N_-^N$ and all $i \in N$,

$$s_i = 0 \implies \varphi(x) \neq i.$$

Positive Unanimity: If all other advisers nominate i, then i is nominated by the nomination rule: for all $x \in N_-^N$ and all $i \in N$,

$$s_i = n - 1 \implies \varphi(x) = i.$$

Monotonicity: If an adviser i is nominated, and everyone but j had nomination profile x_{-j}, then i must be nominated when the strategy played is x_{-j} and in addition j nominates i: for all $i, j \in N$, for all $x_j, x_j' \in N \setminus \{j\}$, and all $x_{-j} \in N_-^{N\setminus j}$,

$$\{\varphi(x_j, x_{-j}) = i \text{ and } x_j' = i\} \implies \varphi(x_j', x_{-j}) = i.$$

Notice that impartiality is our weakening of strategy-proofness. Voter equity will consist of anonymous ballots, no-dummy, and properties introduced later called full-influence and full-pivot. Candidacy equity will be represented by no-exclusion, and a property introduced later called candidate neutrality.

3.3 Impartiality

The first result is an impossibility result, such that for any impartial nomination rule, you need at least $n \geq 4$ agents, specifically as voters.

Proposition 2 (Altman and Tennenholtz (2008))**.** *If $n \leq 3$, there is no nomination rule that satisfies both impartiality and no-exclusion.*

Altman and Tennenholtz (2008). Let $n = 3$. Let $(x_1^1, x_2^1, x_3^1), (x_1^2, x_2^2, x_3^2), (x_1^3, x_2^3, x_3^3)$ be solutions where $\varphi(x_1^i, x_2^i, x_3^i) = i$. By φ being impartial, we have $\varphi(x_1^2, x_2^1, x_3^1) = 1, \varphi(x_1^2, x_2^1, x_3^2) = 2$, and $\varphi(x_1^3, x_2^3, x_3^1) = 3$. Now consider the strategy profile (x_1^2, x_2^3, x_3^1). If $\varphi(x_1^2, x_2^3, x_3^1) = 1$, then by impartiality of $x_1^3 : \varphi(x_1^3, x_2^3, x_3^1) = 1 \neq 3$, in contradiction of above. Similarly, if $\varphi(x_1^2, x_2^3, x_3^1) = 2$, then $\varphi(x_1^2, x_2^1, x_3^1) = 2 \neq 1$, and if $\varphi(x_1^2, x_2^3, x_3^1) = 3$, then $\varphi(x_1^2, x_2^3, x_3^2) = 3 \neq 2$, which forces the contradiction. $\square$

The basic idea is that with impartiality, since your vote cannot affect your own chances of winning, this is the same as only considering other people's vote when determining if you win. With at most three agents, this reduces determining the elected using only two messages, among which a deviation must be consequential enough to violate impartiality. Thus, impartial nomination rules require at least four agents.

3.3.1 *Treatment of Ballots*

With four agents, an impartial nomination rule exists. We now consider if an impartial nomination rule exists that satisfies anonymous ballots. Unfortunately, there is no anonymous and impartial nomination rule that is either a function, Holzman and Moulin (2010), nor a correspondence, Tamura and Ohseto (2014).

Theorem 4 (Holzman and Moulin (2010))**.** *The only nomination rule that satisfies impartiality and anonymous ballots is the constant rule.*

Proposition 3 in Holzman and Moulin (2010). Assume φ satisfies impartiality and anonymous ballots. Since by anonymous ballots only the number of votes matter, then $\varphi(x) = \varphi(s)$. Denote by s_i^{n-1} that the number of votes for agent i is $n-1$, and let $\varphi(s_i^{n-1})$ denote $\varphi(x)$ where x such that $s_i(x) = n-1$.

Let $j = \varphi(s_i^{n-1})$, for some agents i, j. Then, by impartiality, $j = \varphi(s_i^{n-2})$, otherwise, if $j \neq \varphi(s_i^{n-2})$, then agent j could become a loser by changing his vote, a contradiction of impartiality. Therefore $j = \varphi(s_i^{n-2})$. *Notice, by anonymous ballots, this is also true of x' such that $\delta(x') = s_i^{n-2}$ if $x_j = x'_j = i$ and someone else $k \in N_{-\{i,j\}}$ didn't vote for i*, such that $x_k = i$ and $x'_k \neq i$. Then, by impartiality, $j = \varphi(s_i^{n-3})$. Continue recursively, until $j = \varphi(s_i^0)$, which implies that whether everyone but i votes for i, or no one does, and any number in between, that agent j always wins. As a consequence, φ is a constant rule that always picks j.

$\square$

What is going on here? Well, if the nomination rule is impartial, that means agent i's vote cannot affect his own chance of winning. Moreover, if the nomination rule is anonymous, that means his vote cannot affect anyone else's chance of winning either; thus, his vote cannot matter. However, this happens for everyone, thus, all votes are thrown out, and the nomination rule will always end up choosing the same contestant as winner. Many a rigged election would satisfy impartiality and anonymous ballots then. Following the same logic, Tamura and Ohseto (2014) extends the impossibility result to the case where they have a nomination rule that is a correspondence, not just a function. Thus, it is impossible to have anonymous ballots and impartiality.

3.3.2 *Message Space Symmetry*

Interestingly, anonymous ballots here is similar to quasi-symmetry in Altman and Tennenholtz (2008), but his definition makes explicit that everyone's message spaces are identical. In the proof above, each person's message spaces are the same size, but not equal because they cannot vote for themselves. If we asymmetrically restricted message spaces further, we could design a nomination rule that satisfies impartiality and anonymous ballots. However, the nomination rule developed in Holzman and Moulin (2010) that does so, satisfies neither the spirit of voter equity nor candidate equity.

Theorem 5 (Holzman and Moulin (2010))**.** *A median nomination rule with restricted message spaces such that $\emptyset \neq M^i \subseteq N \setminus \{i\}$, and $n \geq 5$, a tree Γ on N with at least two nodes of degree 2 or more, and one node of degree 3 or more, exists that satisfies impartiality, monotonicity, anonymous ballots, no-dummy, and no-exclusion.*

Now the proof is technical, but works by restricting the message space of any agent who is a node of degree greater than 1. For example, if you are the median agent, the mechanism is more likely to select you for nominee, but you also have a smaller message space than the other agents. This allows the mechanism to get around the impossibility results in the earlier proof. Consider the italicized part of the proof: When $j \in \varphi(s_i^{n-2})$, it is not possible to use anonymous ballots to create an equivalence between the two vote profiles where $x_j = i$ and one where $x_j \neq i$ and *someone else* k votes for i, because message spaces are asymmetrically restricted and that someone else k might not exist. Thus, it *technically* satisfies anonymous ballots, but not the spirit of voter equity.

In their theorem they also construct the mechanism, but it is technical and depends on the number of agents, and the shape of the tree Γ on N. Instead, we introduce an example: Let there be five agents, and let agent 3 have three degrees, agent 4 have two degrees, and all other agents have one degree. This requires Message spaces as $M^1 = \{2, 3, 4, 5\}, M^2 = \{1, 3, 4, 5\}, M^3 = \{4, 5\}, M^4 = \{1, 2, 3\}$, and $M^5 = \{1, 2, 3, 4\}$. The Median Agent is 3, and the next most median agent is 4. Notice that any agent can win: Agent 1 wins if nominated by 2,4, and 5. Agent 5 wins if nominated by 1,2, and 3. Further notice that agent 3 is the only agent that all others can vote for and, as the median agent, if agent 1 and 5 vote for 2, agent 2 and 3 votes for 4, and agent 4 votes for 3, with votes being $(2, 4, 4, 3, 2)$, then agent 3 is the nominee despite receiving the fewest votes. Thus, the mechanism requires bias in the treatment of candidates even while restricting message spaces.

The median nomination rule's drawbacks include that in order to satisfy impartiality, it biases selection towards the median agent, sometimes drastically, and in order to satisfy anonymous ballots, it *restricts* some voters message spaces differently from others. If you wanted anonymous ballots for voter equity – then the median voter rule fails in that respect. If asymmetrically restricting message spaces is undesirable, the impossibilities of a completely unrestricted message space are worse, as seen in Mackenzie (2018).

In Mackenzie (2018), he considers two different nomination rules used in Papal elections, the Gregorian rule and the Piusine Rule. A **Gregorian** nomination rule has a 66% supermajority threshold, $M^i = N \setminus \{i\}$, and in the event no one wins, then a recasting of ballots is done. A **Piusine** nomination rule has a 66% supermajority threshold, $M^i = N$, and in the event no one wins the vote, a recasting of ballots occurs. Notice the Piusine nomination rule has unrestricted message space.

Additionally, let us define a property called **candidate neutrality** such that if we permute contestants on the ballots, then the nominee is similarly permuted. First, let S_N be the set of all permutations on N, and let $\sigma \in S_N$, be such a permutation.

Definition 8 (Candidate Neutrality)**.** *For each $x \in M^i$, and each $\sigma \in S_N$, if $\sigma(x) \in M^i$ and $\varphi(x) \in N$, then $\varphi(\sigma(x)) = \sigma(\varphi(x))$.*

With candidate neutrality we can see the problems created for the Piusine nomination rule when it allows self-nomination, which the Gregorian nomination rule does not suffer.

Theorem 6 (Mackenzie (2018))**.** *A Piusine nomination rule satisfies impartiality and:*

1. Anonymous ballots *if and only if it is a constant nomination rule.*

2. Candidate neutrality *if and only if the nomination rule always recasts ballots, $\varphi(x) = \{$Recast Ballots$\}$ for any $x \in N^N$.*

Intuitively, when a nomination rule allows self-nomination like in the Piusine format, then that nomination rule cannot use the score of the vote to determine the nominee. If it did, then an agent could always change their vote to himself if they were within one vote of the threshold to win. Thus, the only nomination rule that is Piusine, impartial, and anonymous is the constant rule, where nobodies' vote matters. If we look at Piusine, impartial, and candidate neutrality, a nomination rule does not exist. Thus, an impartial and practical Piusine Rule does not exist. However, an impartial Gregorian Rule does exist, and satisfies many desirable properties.

Theorem 7 (Mackenzie (2018))**.** *The Gregorian nomination rule satisfies impartiality, monotonicity, positive unanimity, anonymous ballots, and candidate neutrality.*

Notice, that the Gregorian rule also satisfies negative unanimity, and that candidate neutrality implies no-exclusion, such that the Gregorian nomination rule has many strong properties. Unfortunately, this is possible only because of the recasting of ballots. What recast looks like in practice is simply that the agents are locked in the building, and recast ballots, which can be the same vote as previous, until a consensus reached. Thus, it is possible that the electors are imprisoned, recasting ballots, for years. This fact undermines its universal applicability, but it is necessary, theoretically. If a nomination rule does not allow recasting of ballots, it is called **decisive**. A decisive Gregorian or Piusine Nomination rule would satisfy the premises of the impossibility result in Holzman and Moulin (2013), discussed in the next section. Further, if the decisive Gregorian rule only satisfies impartiality and candidate neutrality, then it has to be a dictatorship. Thus, with impartiality and candidate neutrality, we lose all voter equity.

Theorem 8 (Theorem M1 in Mackenzie (2018))**.** *A Gregorian nomination rule satisfies impartiality, decisiveness, and candidate neutrality if and only if it is a dictatorship scrutiny:* $\varphi(x) = x_i$ *for any* $x \in N_{-}^{N \setminus i}$.

Therefore, the Gregorian rule must allow recasting of ballots, and without recasting, it results in a dictatorship. Thus, when it is not possible to lock the electors in a building for a couple of years, we should substitute another nomination rule for the Gregorian nomination rule. And, unlike the Piusine nomination rule, we consider message spaces without self-nomination. Thus, from now on, all message spaces are $M^i = N \setminus \{i\}$.

3.3.3 *Negative Unanimity*

Instinctively, we believe a person whom no one voted for should not be nominated. If a nomination rule satisfies negative unanimity, then a person no one nominated cannot become the nominee. This simple property, however, turns out to be difficult to satisfy in practice.

Theorem 9 (Holzman and Moulin (2013))**.** *There exists no nomination rule that satisfies impartiality, positive unanimity, and negative unanimity.*

Since monotonicity, impartiality, and no-exclusion imply positive unanimity, Theorem 4 also implies there exists no nomination rule that satisfies impartiality, no-exclusion, monotonicity, and negative unanimity.

Corollary 1 (Holzman and Moulin (2013))**.** *There exists no nomination rule that satisfies impartiality, no-exclusion, negative unanimity, and monotonicity.*

For illustration, let us look at some intuitive nomination rules, such as plurality rule, and see which properties they fail. Since the plurality rule fails impartiality, we consider the case where there is a default agent i^* selected when there is no clear nominee by plurality. To define these rules, let $s_i(-j,k)$ be the score of agent i in $N \setminus \{j,k\}$.

Definition 9 (Plurality with Default)**.** *Plurality with Default, Plu*$^{i^*}(x)$*:*

If for some $i \neq i^*$, $s_i(-j,i^*) > s_i(-i,i^*)$ *for all* $j \neq i^*\}$*, then Plu*$^{i^*}(x) = i$*, otherwise Plu*$^{i^*}(x) = i^*$.

Notice, it fails no-dummy because i^*'s vote is never counted, fails negative unanimity because i^* could win even without receiving votes, and even fails monotonicity because agent

i^* can go from winning with zero votes, to losing by gaining a vote, when this breaks a tie to create a plurality winner. Thus, while this satisfies impartiality, it violates other highly desirable properties. So let us transform it into a majority rule with default.

Definition 10 (Majority with Default)**.** *Majority with Default, Maj*$^{i^*}(x)$*:*

$$\textit{If for some } i \neq i^*, s_i(-i^*) \geq [\![\frac{n}{2}]\!], \textit{ then Maj}^{i^*}(x) = i, \textit{ otherwise Maj}^{i^*}(x) = i^*.$$

Again, this mechanism fails no-dummy and negative unanimity. However, it now satisfies monotonicity. Notice, that this mechanism is similar to the Gregorian Rule from Mackenzie (2018), except it has a default agent i^* instead of the recasting of ballots. As the Gregorian rule had many desirable properties, the Majority rule will also, but being decisive and using a default agent, we lose no-dummy and candidate neutrality. Next, we transform the default agent into a "Default-Maker" who chooses the nominee in case of a failed majority rule, rather than being the nominee himself.

Definition 11 (Majority with Default-Maker)**.** *Majority with default-maker, Maj*$_{i^0}(x)$*:*

Given that $x_{i^0} = j$*, if for some* $i \neq i^0, j$*, such that* $s_i(-j) \geq [\![\frac{n}{2}]\!]$*, then Maj*$_{i^0}(x) = i$*, otherwise Maj*$_{i^0}(x) = j$*.*

Now we satisfy negative unanimity, but it fails to satisfy no-exclusion, because the default-maker i^0 can never win. Since it fails No-Exclusion, it fails positive unanimity. Further, agent j is a dummy. Thus, when there is no nominee that clears the majority threshold, we can either have a default agent i^* which is biased among candidacies (like a constant rule), or we can have a default-maker i^0 which is biased among the voters (like a dictatorship). Both fail no-dummy.

Unfortunately, there are examples of nomination rules that satisfy only impartiality, no-exclusion, and negative unanimity, but they lose monotonicty[6]. Monotonicity is a necessary property, since we are looking for equilibrium in any nomination rule we use.

[6]An example in the Paper is a modified version of a Median nomination rule from Holzman and Moulin (2010)

In Holzman and Moulin (2013), the authors develop a nomination rule called the partition method. The partition method will satisfy monotonicity, no-dummy, impartiality, and no-exclusion; only failing negative unanimity. It does so by first partitioning the nomination into districts, each person votes once, possibly for someone outside their district, but the winner will be whoever wins a majority of the votes in their local district, and then has the most votes overall.

The **partition method** proceeds in two steps. Step one, partition the set of agents into multiple groups of at least three agents, except group 1 having at least four agents, including i^*, which is the default agent. Each group has a plurality threshold q_k equal to half of one more than the group population[7]. Then, each agent nominates within their group k, and any agent who receives equal to or more nominations than q_k is called a local winner. Step two: Decide the nominee. If there are no local winners, then default agent i^* is the nominee. If there is only one local winner i, then i is the nominee. If there are multiple local winners, then the nominee is any agent who was in a group of only three agents, otherwise, the nominee is the local winner who received the most votes when local winners' votes excluded, ties broken by being in the lowest group number.

With the partition method established, we introduce full-influence to capture voter equity, while avoiding the impossibility results of the stronger property anonymous ballots. Given nomination rule φ and three agents i, j, and j', we say that agent i is **pivotal** for agents j and j' if, for some profile $x_{-i} \in N_{-}^{N \setminus i}$, there exists $x_i, x_i' \in N \setminus \{i\}$ such that:

$$\varphi(x_i, x_{-i}) = j, \quad \varphi(x_i', x_{-i}) = j'.$$

For any advisers i and j, Adviser i is **influential** for j if there exists some adviser j' such that i pivotal for j and j'. A nomination rule satisfies **full-influence** if all advisers are influential.

Theorem 10 (Holzman and Moulin (2013))**.** *The partition method satisfies impartiality, full-*

[7]Exception being group 1 with the default agent i^*, which is just one half of the group population.

influence, and monotonicity.

Holzman and Moulin (2013). The proof proceeds by showing first that the partition method satisfies impartiality, then full-influence, and finally monotonicity. For impartiality, notice that by the definition of the thresholds of the local elections, at most one agent in each district can win. Suppose some agent i, who is not the default agent i^*, wins the prize. He cannot change his vote to cease being the local winner, nor can his vote affect other local winners since local winners determined only by the vote in their district, and finally, his vote cannot determine the winner from the local winners since the local winners' votes are not considered in step two. Thus, he wins despite his alternatives. Now suppose the winner is i^*. If i^* is a local winner, than the previous argument applies. If i^* won by default because there were no local winners, then his vote does not count in step one. Therefore, the partition method is Impartial.

To verify the partition method satisfies full-influence, construct the following: Let i and j be from the same district, and let j' be from another district. Now let the nomination profile of all other contestants than i be x_{-i} such that j' is a local winner, and the $s_j \geq s_{j'}$ and j is not a local winner without i's vote. Thus, if $x_i = j'$, then the winner is j', and if $x_i = j$ then j becomes a local winner and has more votes than j' and becomes the winner.

To finally prove monotonicity, let i be the winner, and now receives an additional vote from j. If $i = i^*$ won by default, then receiving another vote could at most make him the only local winner, making him win the prize regardless. If i was already a local winner before j's vote, than he would remain a local winner. If j's vote does not affect the other local winners, then i clearly still wins the prize. If j's vote costs j' a local election such that they are no longer a local winner, then their vote could be cast for a competitor of i, but since i also receives the additional vote of j, i still wins the prize. Therefore, the partition method satisfies monotonicity, full-influence, and impartiality. $\square$

Notice that in the partition method, the default agent can win without any votes. This violates negative unanimity. If there are many districts for voting, however, this is unlikely, and it is uncertain if negative unanimity is important to have. Further, the partition method does not satisfy **full-pivots**, where for any three agents i, j and j', i is pivotal for agents j and j'. For example, consider when i is a different district from j and j', who are in the same district, and j' is the local winner. No change in x_i would be able to be help j become the local winner. Thus, i would not be pivotal for j and j'. Again, we do not find this to be a very big problem, because the fix, the cross-partition method, is a much more complex mechanism requiring two additional partitions of agents in addition to districts.

An example given in paper to understand the cross-partition method: Let $n = 14$, and create three districts $N_1 = \{1, 2, 3, 4, 5\}$, $N_2 = \{6, 7, 8, 9\}$, and $N_3 = \{10, 11, 12, 13\}$, and let 14 be the default agent i^*. The agents in each district are further partitioned by two attributes: age and gender. See Table:

N_1	Male	Female	N_2	Male	Female	N_3	Male	Female
Young	1,2	3	Young	6	7	Young	10	11
Old	4	5	Old	8	9	Old	12	13

Table 1: Cross-partition method example, reprinted from Holzman and Moulin (2013)

Using the table above, district k, one partition is into ages (rows), and the other into genders (columns). The partitions of the districts are orthogonal because no row and column is empty, and this is necessary. Now, to be nominated by the cross-partition method, it is done in three ways, either by being an outer-hero, an inner hero[8], or by being default agent 14 when no one else elected. For an agent in N_1 to be an outer-hero, he needs to receive the votes from an entire age group in N_2, and an entire gender group in N_3, such as 6, 7, 11, and 13. If this agent is the only outer-hero, he is elected even if there are inner-heros. If there is another outer-hero, it is decided by a tiebreaker. For an agent in N_1 to be an inner-hero, he needs to receive all of the other votes in his district, such as 2, 3, 4, and 5. If Agent 1 is an inner hero and there are no outer-heroes, he would be the cross-partition method's nominee if he is the only inner-hero or wins the tiebreaker.

[8]Similar to local hero in partition method.

Thus works the cross-partition method, more complex than the partition method, but, by Theorem 2 of Holzman and Moulin (2013), the cross-partition method satisfies impartiality, monotonicity, and full-pivot. Therefore, by accepting greater complexity, we could upgrade full-influence to full-pivot. Neither partition method satisfies negative unanimity either. Yet, before we introduce a nomination rule that satisfies impartiality and negative unanimity, we first introduce a restatement of the impossibility result.

Theorem 11 (Alon et al. (2011))**.** *Let $n \geq 2$ and $k \in 1, ..n-1$. Then there does not exist a k-selection nomination rule that satisfies impartiality and negative unanimity.*

Now first off, a k-selection nomination rule means it selects a set of nominees of size k. Thus, if $k = 2$, then the nomination rule always selects two nominees. The proof shows that the impossibility is because of impartiality, negative unanimity, and the fixed size of the set of nominees. Thus, by making the size of the set of nominees endogenous, Tamura and Ohseto (2014) are able to get positive results with a plurality rule with runners-up.

Let $F_x = \{i \in N : s_i(x) \geq s_j(x), \text{ for all } j \in N\}$, denote the set of all agents that who received the most votes. Then the **plurality with runner-ups** is

$$\varphi(x) = \begin{cases} \{j\} \cup \{i \in N : x_i = j \text{ and } s_i(x) \geq s_j(x) - 1\} & \text{if } F_x = \{j\} \\ F_x & \text{otherwise.} \end{cases}$$

Theorem 12 (Tamura and Ohseto (2014))**.** *Let $n \geq 4$. Then the nomination rule* plurality with runner-ups φ *satisfies impartiality, positive unanimity, and negative unanimity.*

Tamura and Ohseto (2014). Clearly, since this is a plurality rule, it satisfies positive unanimity. To prove it satisfies negative unanimity, assume i received zero votes in x. Then someone other agent j received at least two votes, making $s_j(x) - 1 > s_i(x)$. Therefore, i not nominated.

Finally, to see that it satisfies impartiality, consider $i \in \varphi(x)$[9].

[9]Which means we can also assume i always has at least one vote less than the most.

Case 1: $i \in F_x$. Let $k \in N \setminus \{i\}$, such that $x'_i = k$. If $k \in F_x$, then $\{k\} = F_{(x'_i, x_{-i})}$, and since $x'_i = k$, and i has one vote less, $i \in \varphi(x'_i, x_{-i})$. If $k \notin F_x$, then $s_k(x) < s_i(x)$ such that $s_k(x'_i, x_{-i}) \leq s_i(x'_i, x_{-i})$. Therefore, $i \in F_{(x'_i, x_{-i})}$ and thus $i \in \varphi(x'_i, x_{-i})$. Therefore, i who had among the most votes, would still be nominated no matter whom he votes for.

Case 2: $F_x = \{j\}$ and $x_i = j$. Let $k \in N \setminus \{i, j\}$, such that $x'_i = k$. If agent k had one less vote than j under x, then $\{k\} = F_{(x'_i, x_{-i})}$, i would have one less vote than k, and by voting for k, would also be elected, $i \in \varphi(x'_i, x_{-i})$. If agent k had 2 or more votes less than j, then in (x'_i, x_{-i}), agents i and j have the same and most votes, $\{i, j\} \subseteq F_{(x'_i, x_{-i})}$, such that $i \in \varphi(x'_i, x_{-i})$. Therefore, if i is a runner-up and winner, his change of vote does not affect his own nomination status. Thus, the plurality rule with runner-ups is impartial too. $\square$

In Tamura and Ohseto (2014), they are able to get a nomination rule that satisfies all the properties for Theorem 4 in Holzman and Moulin (2013), by nominating a *set of nominees* of endogenous size. In Tamura and Ohseto (2014) they use a plurality rule where anyone who is close enough in nominations to the front-runner is called a “runner up” and is also nominated. The plurality rule with runner up satisfies impartiality, positive unanimity, negative unanimity, and even full-pivot, although full-pivot is not shown in the paper. It could also nominate everyone, which defeats the purpose of the entire voting process. There are many economic environments where the size of the set of nominees is significant: There can only be one president, one gold medalist, and two prom royalty. A further problem with the plurality rule with runner-up is that all votes must be traceable to their voter. In short, ballots can not be secret. As we can see in such historical cases as Tammany Hall in New York city circa 1860, this allows parties to coerce the vote. Thus, it is an interesting nomination rule, but maybe less useful in real life than the original partition method or the modified majority rules.

impartiality, much like Strategy-proofness, creates difficulties for implementation. With default-agent nomination rules in Holzman and Moulin (2013), a default contestant will win more often, similar to an incumbent in an election. Whereas in super-majority nomination with re-voting, like in Mackenzie (2018), the nomination can take *years* to conclude, during which time *someone* is the acting nominee. Thus, an impartial nomination rule tends to favor the status quo. Nevertheless, when it does change, it is less likely to be due to manipulation. In a democratic society that runs on the consent of the masses, this status-quo bias could be the tradeoff desired.

We hope that throughout this literature survey we have shown the intuitive trade off that exists when attempting to create a nomination rule that satisfies optimality of simple strategies with the impartiality condition, voter equity with the anonymous ballots, no-dummy, and full-influence, and candidate equity with such properties as no-exclusion and candidate neutrality. Notice that the starkest impossibility was between impartiality and voter equity, with possible research needed to get a clearer picture of how effectively we can trade off contestant equity for voter equity.

Finally, it is worth recognizing the complete impossibility for a nomination rule to be impartial and satisfy negative unanimity, and still nominate a fixed number of nominees, making us question how comfortable we are letting someone win without votes. Thus, we have many options of nomination rules, some better suited to certain circumstances than others are. Since nomination rules tend to become instituted, a further question could be the reciprocal relationship between the nomination rules and the culture that develops from it. With rules that create incumbency, do we see voters become less entrenched in their positions and more willing to vote against their interests? If with non-impartial nomination rules, we see voters who are incapable of the level of strategic thinking not vote, would we want to employ an impartial nomination rule instead? In these models, we treat voter preferences as fixed, but what if the preferences are affected by the culture and the culture is affected by the nomination rule itself? In short, to what extent is a voter's preference in our models not primal, but an intermediate product of the agent's ability to influence their circumstances?

4. (UN)INFORMED CHARITABLE GIVING WITH COMMON VALUES

4.1 Introduction

Should a charity always advertise more? Charities, unlike a business that sells a product, must sell a cause. Where a business engages in sales, a charity instead engages in fundraising. If advertising and sales where free, a business would always do better by advertising more, but the benefit for a charity is less clear. In "(Un)Informed Charitable Giving" by Krasteva and Yildirim (2013), they find more informed donors always increase total donations so the charity would want to advertise as much as possible. However, the authors only consider the case of heterogenous private value public goods and not the case of common value public goods. This paper fills that gap. In contrast, if the charity had a common value public good, the charity would maximize total donations with a mix of informed and uninformed donors and not when all donors informed. Thus, if advertising was free, a charity with a common value public good would still prefer a limited fundraising campaign.

Our paper can be considered an extension of Krasteva and Yildirim (2013) to the common value public good environment. It looks at the value of information for donations to a charity, where a charity is trying to raise donations to finance the construction of a public good. If total donations do not exceed the cost of providing the public good then it will not be provided. Their paper finds that charities should find a way to make information cheap, such as advertising. Other papers in the field consider a signal, rather than cost argument, where the charity can publish a large donation as evidence of the charity's quality.

In Vesterlund (2003), she considers a discrete public good game where quality of the public good is unknown. The charity knows whether it is a high-quality or low-quality charity and can commit to reveal the first donor's donation. No donors observe the charity's type, but the first donor can pay cost $c \geq 0$ to learn the type before they make their donation. Simplifying their results, they get a separating equilibrium, with optimal strategies being stochastic. Thus, we see the high-quality charity more frequently committing to announce the first donation, and the first donor randomizing on acquiring information, conditional on its cost. If the first donor pays for information, and they learn the charity is high quality, they donate a large amount to signal to other donors the charity's quality. However, if they observe otherwise, they donate nothing. Interestingly, if the charity is high quality and the first donor observes this, then the resulting total donations are greater than if the the charity's quality was commonly known.

Andreoni (2006) modifies Vesterlund's model to consider the case where quality is not binary, and the first donor is not exogenously determined. These changes in the model weaken the results in Vesterlund (2003), and Andreoni finds that a leader might not emerge because effective signalling of high quality can be too costly. As a result, in this economic environment, signalling is still effective but increasingly unlikely to occur, creating a role for government grants. However, Andreoni and Payne (2003) shows that government grants result in charities drastically reducing their fundraising activities. Without government funds, and if we assume soliciting is costly, then Correa (2017) shows that we should solicit the wealthiest person first, and Yildirim et al. (2013) shows that the model with costly solicitation is the same as the standard model with donors wealth subtracted the cost of solicitation. Thus, it is appropriate to model the cost of becoming informed as a subtraction from the donors private good, which we do in our model by assuming all donors are uninformed and can pay $c \geq 0$ to become informed, the cost c subtracted from utility. Further, the fact we should solicit the wealthiest person first translates to soliciting the person with the highest private value of the public good first, but in our common value situation, that is not possible. Everyone has the same valuation. Therefore, this paper can not have a lead donor.

A major difference between our paper and the previously mentioned ones is that we assume a subscription public goods game, where donors make pledges that are only collected if the pledged donations are greater than the cost threshold to provide the public good. In Nitzan and Romano (1990), they show that uncertainty about the cost threshold reduces total donations.

McBride (2006) shows that for a distribution that skews towards high values of the public good, the charity benefits from *increased* uncertainty of the threshold cost, because it increases each donors likelihood of being pivotal. Likewise, if the distribution skews low values, then it diminishes the donations. Further, his results indicate that total donations can decrease if too many donors become informed. Unlike his paper, we assume the distribution of the cost threshold is uniform, and that the uncertainty arises from the percentage of the donors who are informed.

The paper proceeds by introducing the model in section 1, then discusses the results from Krasteva and Yildirim (2013) in more detail in section 2, and section 3 is composed of two subsections: the exogenous information case and the endogenous information case.

4.2 Model

A charity is collecting donations from $n \geq 2$ risk-neutral donors to provide a discrete public good. Ex ante, donors are uninformed of the common value v of the public good, but they know it is drawn from a continuous distribution with CDF $F(v)$ and support $v \in [0, 1]$. Denote the mean of F as μ.

Each donor i can learn the value v by paying a fixed cost $c \geq 0$. Agents then make donations based on the information they know, without communication. Let x_i be donor $i's$ donation and $X \equiv \sum_i x_i$ be the total donation.

The public good is provided if and only if the total donations exceed the cost of the public good, $X \geq k$, where k is the cost of providing the public good. At the time of donations, k is unknown to the donors and the charity. The distribution of cost k, is known to be independent of the value of the public good v, and is uniformly distributed in $[0, K]$, with $K > \frac{n}{2}$. The donations are of a subscription nature: if the total donations exceeds the cost k, then the public good is provided and the excess pledges are kept by the charity. However, if the total donations do not exceed the cost k, then they are refunded. If the public good is provided, donor's utility is $v - x_i$, but if the public good is not provided they get a reservation utility of 0.

Thus, the order of play is: (1) charity chooses cost c; (2) a value v of the public good is drawn; (3) donors can pay c to learn the value v; (4) donors pledge their donation; (5) The cost k of the public good is drawn and learned by the charity; (6) if $X \geq k$, then the charity collects donors's donations and provides the public good, otherwise, then the public good is not provided, no donations collected, and everyone receives the reservation value 0. The goal of the charity is to maximize the probability of providing the public good. This is equivalent to maximizing the total donations X in equilibrium. The solution concept is a symmetric Bayesian-Nash equillibrium.

4.2.1 *Existing Results*

In Krasteva and Yildirim (2013), their model is the same as our own, except the public good has private independent values, v_i, for each donor i. As their values are private, informed donors only know their own v_i and are ignorant of other donors value's v_j. Uninformed donors are ignorant of everyone's valuation, including their own. In Krasteva and Yildirim (2013) their charity is trying to maximize total donations. Let θ denote the expectation that other agent's are informed.

Proposition (Proposition 2 in Krasteva and Yildirim (2013))**.** *In equilibrium, both* $\bar{x}^I(\theta)$ *and* $x^U(\theta)$ *are strictly decreasing in* θ *while* $\bar{X}(\theta)$ *is strictly increasing in* θ*.*

Here, $\bar{x}^I(\theta)$ is individual expected informed donations, $x^U(\theta)$ is individual uninformed donations, and $\bar{X}(\theta)$ is expected total donations. Thus, Proposition 2 says that we expect both informed donors and uninformed donors will free ride more as the expected share of informed donors, θ, increases. However, total donations still strictly increase in θ, because the increased free-riding is offset by the gains from uninformed donors becoming informed donors, who donate more on average. Thus, total donations are maximized when donors are all informed.

Now, when Krasteva and Yildirim (2013) look at the endogenous information case, where donor's can pay to become informed at cost c, they find that the value of information is always positive but is decreasing as donors become increasingly informed, and consequently, the value of information is decreasing as other's expected total donations are increasing. Figure 1 below depicts an example of the value of information: positive but decreasing towards 0. Thus, any information cost $c > 0$, would result in a second best solution because the donors would not become fully informed.

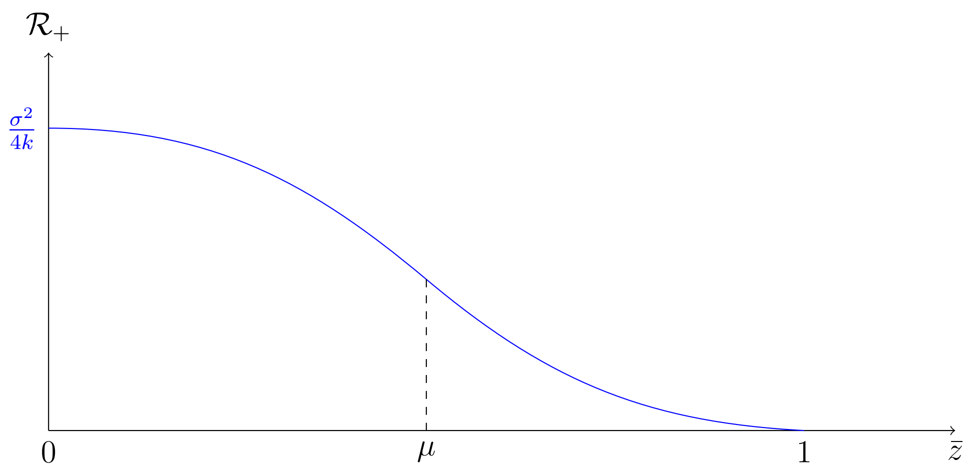

Figure 1: The value of information, reprinted from Krasteva and Yildirim (2013)

Thus, for the charity to maximize total donations they want information to be free, $c = 0$. In contrast, our model with the common value public good would maximize total donations when information is costly.

4.3 Results

4.3.1 *Benchmark Case: Exogenous Information*

Suppose that with a fixed probability, θ, each person privately knows the public good common value v, while with probability $1 - \theta$, they are uninformed. Let $x^I(v, \theta)$ denote an informed donation, and $x^U(\theta)$ an uninformed donation, and $x(v, \theta) = \theta x^I(v, \theta) + (1 - \theta)x^U(\theta)$ be the unconditional donation. Thus, the expected utility is given by:

$$
\begin{aligned}
u_i^I(x_i, v) &= (v - x_i) * Pr\{x_i + \textstyle\sum_{j \neq i} x_j(v, \theta) \geq k\} \\
&= (v - x_i) * E\{\frac{x_i + \sum_{j \neq i} x_j(v,\theta)}{K}\} \\
&= (v - x_i) * E(\frac{x_i + \theta(n-1)x^I(v,\theta) + (1-\theta)(n-1)x^U(\theta)}{K})
\end{aligned}
$$

Now if we maximize the informed donor's utility with respect to their donations x_i, we get:

$$
x^I(v, \theta) \quad = max\{0, \frac{v - \tilde{x}(\theta)}{2 + \theta(n-1)}\} \tag{1}
$$

where $\tilde{x}(\theta) = (1 - \theta)(n - 1)x^U(\theta)$ is the **total other's uninformed donations**. Looking at the equation, we can tell that informed donations go up as the common value v increases, and decreases when total other's uninformed donations increase.

If we repeat the same analysis for the uninformed donations, then we must take the expected value since the common value v is unknown. This gives uninformed donations $x^U(x_i) \equiv E[x^I(v, \theta)] = \int_0^1 (v - x_i) * \frac{x_i + \theta(n-1)x^I(v,\theta) + (1-\theta)(n-1)x^U(\theta)}{K} dF(v)$[10]. However, unlike in the private value's case where uninformed donations equals the informed donations at $v = \mu$, that is not true for the common value case. Because, with common value public good, any uninformed donor i knows $P(X_{-i} \geq k)$ is increasing in their valuation v_i since $v_i = v$. Thus, with a common value public good, because donor i knows the probability that a public good is provided is greater when the value of the public good rises, donor i will overbid less frequently than in the private value case.

Expected informed donations, $\bar{x}^I(\theta)$, is the ex ante expectation of informed donations before v is observed. We find expected informed donations are always greater than uninformed donations, and the equilibrium is unique.

Proposition 1.

[10]Notice that this is different from the private values case, because where I have $x^I(v, \theta)$, the private values case substitutes $\bar{x}^I(\theta)$, since no donor knows other donor's values in the private value case.

For each $\theta \in [0,1]$, there is a unique equilibrium, and it satisfies:

$$\bar{x}^I(\theta) \geq x^U(\theta), \text{ with } \bar{x}^I(1) = x^U(1),$$

where $\bar{x}^I(\theta)$ and $x^U(\theta)$ can be expressed as:

$$\begin{aligned}
\bar{x}^I(\theta) &= \tfrac{1}{2+\theta(n-1)}[\textstyle\int_{\tilde{x}(\theta)}^1 1 - F(w)dw] \\
x^U(\theta) &= \tfrac{1}{2(n+1)}[2\textstyle\int_0^1 1 - F(v)dv - \theta(n-1)\int_0^{\tilde{x}(\theta)} F(v)dv].
\end{aligned}$$

Thus, Proposition 1 in this paper and in Krasteva and Yildirim (2013) are similar results. expected informed donations are greater than uninformed donations, since informed donors can avoid the winner's curse, while uninformed donors must consider the possibility that donors over-donate. Both have unique and well-defined solutions for expected informed donations and uninformed donations for the equilibrium.

Above, we have considered how donations respond to changes in the value v, let us now consider how individual and total donations change with respect to changes in the percent of donors informed, θ. I define $\bar{X}(\theta)$ as the **expected total donations**.

Proposition 2.

1. *$x^U(\theta)$ is minimized for some $\theta' \in (0,1)$.*

2. *$\bar{X}(\theta)$ is maximized for some $\theta^* \in (0,1)$.*

3. *Total Donations are minimized at $\theta \in \{0,1\}$.*

Thus, Proposition 2 diverges from the private value case in Krasteva and Yildirim (2013). Where in the private value case we see that individual donations where always decreasing in θ, here we have uninformed donations increasing for a range of θ, depending on what the distribution of values $F(v)$ looks like. For example, for the uniform distribution, $F(v) = v$, uninformed donations are decreasing for $\theta < \frac{1}{3}$, and increasing thereafter, as can be seen in

Example 1. While it doesn't appear the donors are free-riding, they actually are free riding as θ increases, but that effect is overpowered by the positive effect of the reduced winner's curse from the increasingly pivotal informed donations.

Another divergence for the common value public good compared to the private value public good is that expected total donations are maximized for a partially-informed equilibrium. In the private value case, Krasteva and Yildirim (2013) show that $\theta = 1$ maximizes expected total donations, while in the common value case expected total donations are maximized for a θ that is strictly between 0 and 1. Below is an example of this behavior, where the number of donors is 2.

Example 3. *Let $F(v) = v$ be uniform distribution, and let the number of donors be $n = 2$, then plugging in values, we find that:*

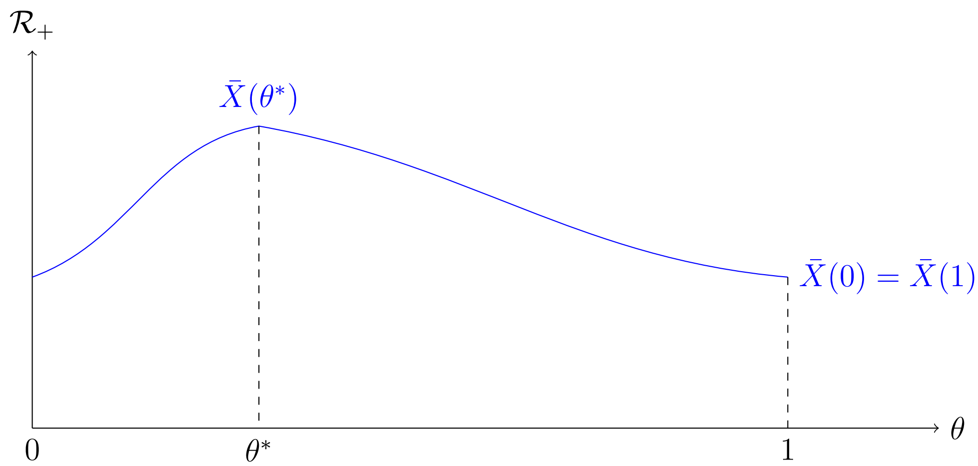

Figure 2: Expected total donations at $n = 2$.

$\bar{X}(0)$ = $n * x^U(0) = \frac{n}{2(n+1)}$ = *0.3333*

$\bar{X}(\frac{1}{3})$ = $n(\frac{1}{3}\bar{x}^I(\frac{1}{3}) + \frac{2}{3}x^U(\frac{1}{3}))$ = *0.3347*

$\bar{X}(1)$ = $n * \bar{x}^I(1) = \frac{n}{2(n+1)}$ = *0.3333*

This example assumes that the distribution is $F(v) = v$, and that the number of donors is $n = 2$. Interpreting the results from the graph: the two endpoints of expected total donations are the same and minimums, $\bar{X}(0) = \bar{X}(1)$, and it is single peaked at θ^*. We can also see it has a couple of inflection points and, while not monotonic, is composed of two simple monotonic halves: expected total donations are growing until θ^* and shrinking thereafter. The max is slightly larger than one third, $\theta^* \approx \frac{1}{3}$, and the values are given below the graph. The total increase in donations at the θ^* compared to fully informed is roughly 4.2%. Thus, if you are a charity receiving one billion dollars in donations at fully informed equilibrium, you would make forty-two million dollars more if fewer people where informed, plus the savings from spending less on fundraising.

4.3.2 *Endogenous Information Case*

Now we consider the case where donors can pay $c \geq 0$ to learn v, and thus whether a donor is informed or uninformed is a choice. Because the public good's common value creates greater awareness of the other donor's strategies, I define the indirect utilities as dependent on θ, $U^I(\theta)$ and $U^U(\theta)$. Formally:

$$U^I(\theta) = E(\max_{x_i}(v - x_i)(\tfrac{x_i+(n-1)*x(v,\theta)}{K})) \tag{2}$$

and

$$U^U(\theta) = \max_{x_i} E((v - x_i)(\tfrac{x_i+(n-1)*x(v,\theta)}{K})), \tag{3}$$

and let the value of information for each donor be the difference between the two indirect utilities:

$$\Delta(\theta) \equiv U^I(\theta) - U^U(\theta). \tag{4}$$

If we find the slope of the Indirect utilities, applying the envelope theorem, we get:

$$\begin{aligned} U^{I'}(\theta) &= \tfrac{n-1}{K} \int_0^1 (v - x^I(v,\theta)) \tfrac{\partial(\theta x^I(v,\theta)+(1-\theta)x^U(\theta))}{\partial\theta} dF(v) \\ \text{and } U^{U'}(\theta) &= \tfrac{n-1}{K} \int_0^1 (v - x^U(\theta)) \tfrac{\partial(\theta x^I(v,\theta)+(1-\theta)x^U(\theta))}{\partial\theta} dF(v). \end{aligned}$$

Information is always positive in value. However, it is less clear if the value of information is always shrinking, since total donations are non-monotonic as θ increases. Thus, if we apply the envelope theorem, we get the slope of the value of information:

$$\Delta'(\theta) = -\tfrac{n-1}{K} \int_0^1 (x^I(v,\theta) - x^U(\theta)) \tfrac{\partial(\theta x^I(v,\theta)+(1-\theta)x^U(\theta))}{\partial\theta} dF(v). \tag{5}$$

Looking at the slope of the value of information, it looks like it is non-monotonic. Lemma 1 proves the slope is non-monotonic. Figure 3 gives an example for the uniform case to show it more explicitly.

Lemma 1.

- $\Delta(\theta) > 0$*, for* $\theta \in [0,1]$*, with* $\Delta(0) > \Delta(1)$*, and* $\Delta'(1) > 0$*.*
- $\Delta(\theta)$ *is strictly decreasing in* K*.*

Thus, according to Lemma 1, because information is always valuable, we know that $U^I(\theta) > U^U(\theta)$, while the charity's total donations are maximized at $\theta^* < 1$. Since, information most valuable at fully uninformed and fully informed, and growing at $\theta = 1$, we know the value of information is non-monotonic, with the value of information minimized between 0 and 1. Finally, if the cost of the public good is increasing, then the value of information is decreasing, because when a project is not completed, both uninformed and informed have the same reservation value, 0.

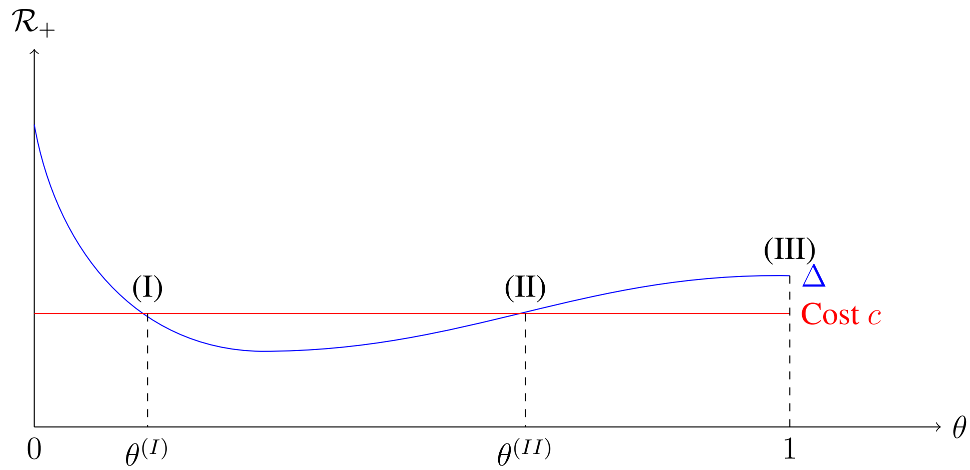

Figure 3: Example: the value of information in uniform distribution

Figure 3 presents an example of how the value of information changes with θ. Figure 3 begins by seeing the value of information quickly fall, then increasing slightly thereafter. The value of information is maximized at $\theta = 0$; and at $\theta = 1, \Delta(1) > 0$.

Finally, the horizontal line "cost c" is cost of information. A donor would benefit to acquire information if the curve for Δ is above the horizontal line c. Thus, if the donors started at $\theta = 0$, then the share of informed would converge to $\theta^{(I)}$, and would be stable there. At $\theta^{(II)}$, the equilibrium would be unstable, and is liable to converge to the stable equilibrium at $\theta = 1$.

Notice further, that if $c = 0$, then the only equilibrium is fully informed, and if the cost $c > \Delta(0)$, then $\theta = 0$ is the equilibrium (and no one pays to become informed). Thus, if we are a charity, and we know that fully informed and fully uninformed are both the minimum expected total donations, then we want to make it costly enough to become informed such that the donors stop at equilibrium θ^*, by setting $c = \Delta(\theta^*)$. In short, with a common value public good, the charity would want information to be costly to maximize its expected total donations.

This intuition extends to the general case, and we get the following proposition.

Proposition 3. *For each cost c, the set of endogenous equilibria $\Theta(c)$ would be:*

for $c \geq \max_\theta \Delta(\theta)$**,** *fully uninformed is the only equilibrium in* $\Theta(c)$*.*

$$\bar{X}(0) = \frac{n}{n+1}[\int_0^1 1 - F(v)dv]$$

for $c < \min_\theta \Delta(\theta)$**,** *fully informed is the only equilibrium.*

$$\bar{X}(1) = \frac{n}{n+1}[\int_0^1 1 - F(v)dv]$$

for $\max_\theta \Delta(\theta) > c > \min_\theta \Delta(\theta)$**,** *there are partially-informed equilibria $\theta^c \in \Theta^c$, where $\theta^c \in \Theta^c$ solves $\Delta(\theta^c) = c$.*

$$\bar{X}(\theta^c) = \tfrac{n\theta^c}{2+\theta^c(n-1)}[\int_{\tilde{x}(\theta^c)}^1 1 - F(v)dv] + \tfrac{n(1-\theta^c)}{2(n+1)}[2\int_0^1 1 - F(v)dv - \theta^c(n-1)\int_0^{\tilde{x}(\theta^c)} F(v)dv]$$

Thus, with the above proposition we can always calculate the equilibrium or equilibria possible for any cost c. If we recall from Proposition 2, there is an optimal $\theta^* \in (0,1)$ that maximizes expected total donations. Thus, if information is free, we get the fully-informed equilibrium, which has less expected total donations than at the partially-informed equilibrium at θ^*. Therefore, we can increase expected total donations by increasing cost c. Further, notice that if $c < \Delta(0)$, then fully uninformed is not an equilibrium, and if $c > \Delta(1)$, then fully informed is not an equilibrium, and thus we know because $\Delta(0) > \Delta(1)$ that there is a cost c such that there is Θ^C that includes neither end point.

Proposition 4. *For any cost $c' \in [0, \min_\theta \Delta(\theta))$, there exists a cost $c'' > c'$ such that $\bar{X}(\theta^{c''}) > \bar{X}(1)$, where $\theta^{c''} \in [0,1]$ satisfies $\Delta(\theta^{c''}) = c''$.*

Thus, with only continuous distribution of the common values of the public good, information being too cheap could lead to lower expected total donations. If information is too easy for the public to acquire, then the charity could increase its donations by restricting the information and making its acquisition more costly. Therefore, if a charity has a common value public good and is running a widespread advertising and fundraising campaign, it might actually increase its donations by advertising less. Intuitively, this works because if the share of the informed donors is small enough, they donate more, because each is more pivotal. But as more donors become informed, they donate less and free-ride more because they have individually become less pivotal. This is similar to the idea in McBride (2006), where higher value donors donate more in greater uncertainty because they become more pivotal. Thus, for a certain range of public goods, extensive fundraising campaigns that would be ill-advised.

4.4 Conclusion

Too many donors being informed can result in lower expected total donations, like in Vesterlund (2003) and McBride (2006). Furthermore, as in McBride (2006), a charity maximizes its expected total donations by increasing uncertainty, in this paper by ensuring a mix of informed and uninformed donors. To do this, the charity will want information to be costly; not too much, not too little, but costly all the same. Thus, our results diverge from Krasteva and Yildirim (2013), where the heterogenous private values case maximizes expected total donations when all donors are informed. Therefore, in the common value case, the crowding out of fundraising activities from government grants seen in Andreoni and Payne (2003) can become desirable, because charities could be fundraising too much. Thus, this paper challenges our general economic intuition that more information is always better.

Future research would be an empirical testing of the paper. The result is an interior mix of informed and uninformed donors, and it would be interesting to see if these results still hold up with the bounded rationality that real life humans experience. The experiment would have a subject play the charity, and the rest of a group play the donors. The charity can choose the cost of acquiring information. I expect subjects would over-purchase information, requiring the charity player to raise the cost more than in the theory. The difference between the theoretical optimal cost and the empirical optimal cost would be capturing the subjects uncertainty aversion. But only testing it in the lab could prove this.

5. CONCLUSION

This dissertation has looked at the problem of information asymmetry in three essays. In the first two essays, information asymmetry is a market failure, and in the last essay information asymmetry is not a market failure but a partial solution to the public good market failure. The first two essays have a mechanism designer and agents, whose participation is assumed. In Section 2, the first essay looks at the problem when the designer must elicit the true ranking of contestants from two biased jurors. Since neither the designer nor any third party ever know if the jurors report a ranking that is not the true ranking, we can only get Nash implementation by reducing the information asymmetry or reducing the domain of preferences. In the essay, we do both using a weak restriction of impartiality called impartial pairs. Three assumptions would prove necessary, the first and third restrict preferences and the second assumes the designer knows the impartial pairs. First, we assume each pair of contestants is an impartial pair for some juror. To reduce the information asymmetry, we assume for the second assumption that the designer knows sufficient impartial pairs he can verify the other two assumptions are satisfied. For the third assumption, we assume the impartial pairs for the two jurors induce overlapping lower contour sets of reported rankings. With all of these assumptions satisfied, there exists a mechanism that Nash implements the true ranking in truth-telling strategies. In fact, the assumptions will prove both necessary and sufficient, such that satisfying anything less than them will not allow a mechanism that Nash implements the true ranking.

In Section 3, we now have three or more agents, but instead of trying to elicit the true ranking, its an election. In the essay, impartiality is a property of the mechanism. The mechanism is a nomination rule, so an impartial nomination rule is one where a agent's vote does not effect their own election. In this manner, impartiality is a weaker condition than strategy-proofness. Thus, this essay surveys the literature of impartial nomination rules, mechanisms with more complex strategies than in Section 2 but the domain of preferences are unrestricted. However, impartial nomination rules can not have anonymous votes, nor can they have equal treatment of each agent's chance of being elected, except for the undesirable constant rules or dictatorship rules. As a consequence, an impartial nomination rule has to have some biased treatment of the agents. The most viable impartial nomination rules have status-quo bias, like the majority rule with default agent.

Finally, we look at a case where information asymmetry actually reduces the consequence of a market failure. Section 4 is a common value public good game, and the agents can choose to be either informed donors or uninformed donors, depending on whether they pay to learn the value of the public good. Therefore, in this public good game, there would be no information asymmetry among the donors if they where all informed or all uninformed. However, the charity wants to maximize the expected total donations, which occurs for a mix of informed and uninformed donors. Thus, the charity will increase the cost of learning the value of the public good to create an equilibrium with asymmetry of information among the donors.

Thus, while information asymmetry is often a market failure, its solutions carry tradeoffs. The solutions to information asymmetry can often be either nonviable or undesirable, be it restricting the domain of preferences like in Section 2 or losing equal treatment of the agents as in Section 3. In Section 4, we show information asymmetry reduces the welfare cost of another market failure, free-riding. Thus information asymmetry is conditionally a problem, and we have to consider the tradeoffs of solutions for information asymmetry.

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APPENDIX A

PROOFS FOR SECTION 2

This appendix contains the proofs of Lemma 1 presented in Section 2. All other proofs are in the essay body.

Lemma 1. Let $\pi \neq \pi_t$. By way of contradiction, suppose that for every swapped pair $(a, b) \in Z(\pi, \pi_t^*) \cup Z(\pi_t^*, \pi)$, the contestant pair (a, b) is not adjacent in π.

Without loss of generality, choose $(a, b) \in Z(\pi, \pi_t^*)$.

By (a, b) not adjacent in π, then there must be a contestant $c \in N_{-\{a,b\}}$ be between a and c, $p_a^\pi < p_c^\pi < p_b^\pi$. Since c is between a and b in π, and (a, b) is a swapped pair, then either (a, c) or (b, c) is a swapped pair.

Let (a, c) be a swapped pair. By assumption then, (a, c) is not adjacent in π. This implies there is a ranking $c_t \in N_{-\{a,b,c\}}$ that is between a and c. Since c_t is between a and c in π, and (a, c) is a swapped pair, then either (a, c_t) or (c, c_t) is a swapped pair.

Iterate, until we have a swapped pair (c_{n-1}, c_n), except this time, because the set of contestants is finite, there is no further contestant $c_{n+1} \in N_{-N}$ that could be between them. As a consequence, (c_{n-1}, c_n) must be adjacent, a contradiction. Therefore, there must be a swapped pair $(a, b) \in Z(\pi, \pi_t^*) \cup Z(\pi_t^*, \pi)$ such that its adjacent in π.

Since the designer knows for each contestant pair a juror who is impartial over it, this means that if a ranking is different from the true ranking, the designer knows there is a swapped pair that is adjacent in π, and for which juror its an impartial pair. $\square$

APPENDIX B

PROOFS FOR SECTION 4

This appendix contains the proofs for all of the results presented in Section 4. Thus, it contains the proofs in the order of Proposition 1, Proposition 2, Lemma 1, Proposition 3, and Proposition 4.

Proposition 1 (Proposition 1)**.**

1. *Ex Ante informed donation is larger than the uninformed donation:* $\bar{x}^I \geq x^U$.

2. $\bar{x}^I(\theta) = \frac{1}{2+\theta(n-1)}[\int_{\tilde{x}(\theta)}^1 1 - F(w)dw]$.

3. $x^U(\theta) = \frac{1}{2(n+1)}[2\int_0^1 1 - F(v)dv - \theta(n-1)\int_0^{\tilde{x}(\theta)} F(v)dv]$.

4. $x^U(1) = \bar{x}^I(1)$.

Proof. **Proof of Item (1.)**

show $\bar{x}^I \geq x^U$.

$$\begin{aligned}
\tfrac{1}{2+\theta(n-1)}[1 - \tilde{x}(\theta) - \textstyle\int_{\tilde{x}(\theta)}^1 F(v)dv] &\geq x^U \\
\tfrac{1}{2+\theta(n-1)}[1 - \textstyle\int_{\tilde{x}(\theta)}^1 F(v)dv] &\geq x^U \tfrac{2+(n-1)\theta+(n-1)(1-\theta)}{2+\theta(n-1)} \\
\tfrac{1}{2+\theta(n-1)}[1 - \textstyle\int_{\tilde{x}(\theta)}^1 F(v)dv] &\geq x^U \tfrac{(n+1)}{2+\theta(n-1)} \\
\tfrac{2}{2(n+1)}[1 - \textstyle\int_{\tilde{x}(\theta)}^1 F(v)dv] &\geq \tfrac{1}{2(n+1)}[2\textstyle\int_0^1 1 - F(v)dv - \theta(n-1)\int_{\tilde{x}(\theta)}^1 F(v)dv] \\
-2\textstyle\int_{\tilde{x}(\theta)}^1 F(v)dv &\geq -2\textstyle\int_0^1 F(v)dv - \theta(n-1)\int_{\tilde{x}(\theta)}^1 F(v)dv \\
2\textstyle\int_0^{\tilde{x}(\theta)} F(v)dv &\geq -\theta(n-1)\textstyle\int_{\tilde{x}(\theta)}^1 F(v)dv \\
2\textstyle\int_0^{\tilde{x}(\theta)} F(v)dv + \theta(n-1)\int_{\tilde{x}(\theta)}^1 F(v)dv &\geq 0
\end{aligned}$$

Proof of Item (2.)

$$
\begin{aligned}
x^I &= max\{0, \tfrac{v-\theta(n-1)x^I-\tilde{x}(\theta)}{2}\} \\
(2+\theta(n-1))x^I &= max\{-\theta(n-1)x^I, v-\tilde{x}(\theta)\} \\
x^I &= max\{\tfrac{-\theta(n-1)x^I}{2+\theta(n-1)}, \tfrac{v-\tilde{x}(\theta)}{2+\theta(n-1)}\}
\end{aligned}
$$

Notice that $\frac{-\theta(n-1)x^I}{2+\theta(n-1)}$ is greatest at 0, and always less than $\frac{v-\tilde{x}(\theta)}{2+\theta(n-1)}$ when this is positive, and vise versa. Therefore, we can substitute:

$$
\begin{aligned}
x^I &= max\{0, \tfrac{v-\tilde{x}(\theta)}{2+\theta(n-1)}\} \\
E(x^I) &= \textstyle\int_{\tilde{x}(\theta)}^1 \tfrac{v-\tilde{x}(\theta)}{2+\theta(n-1)} dF \\
E(x^I) &= \tfrac{1}{2+\theta(n-1)} \textstyle\int_{\tilde{x}(\theta)}^1 1-F(t)dt
\end{aligned}
$$

Where the last step is derived using integration by parts, and $\bar{x}^I \equiv E(x^I)$.

Proof of Item (3.)

$$
\begin{aligned}
x^U(\theta) &= E(\tfrac{v-\theta(n-1)x^I(\theta)}{2+(n-1)(1-\theta)}) \\
&= \tfrac{1}{A}\textstyle\int_0^1 v-\theta(n-1)x^I(\theta)dF \text{ where } A = 2+(n-1)(1-\theta)
\end{aligned}
$$

using Integration by parts.

$dt = dF(v)$ and $t = F(v)$

$s = v-\theta(n-1)x^I(\theta)$ and $ds = 1-\theta(n-1)\frac{\partial x^I}{\partial v}$

$$
\begin{aligned}
x^U = \tfrac{1}{A} \quad & [(1-\theta(n-1)x^I(1))F(1)-(0-\theta(n-1)x^I(0))F(0) \\
& -\textstyle\int_0^1 F(x)[1-\theta(n-1)\tfrac{\partial x^I}{\partial v}]dv]
\end{aligned}
$$

Cancel $F(0) = 0$.

$$x^U(\theta) \quad = \frac{1}{A}[(1-\theta(n-1)x^I(1))F(1) - \int_0^1 F(x)[1-\theta(n-1)\frac{\partial x^I}{\partial v}]dx]$$

And substituting in the fact that

$$\frac{\partial x^I}{\partial v} = \begin{cases} 0 & v \in [0, \tilde{x}(\theta)] \\ \frac{1}{2+\theta(n-1)} & \text{otherwise} \end{cases}$$

to separate the integral.

$$x^U(\theta) \quad = \frac{1}{A}[(1-\theta(n-1)x^I(1))F(1) - \int_0^{\tilde{x}(\theta)} F(x)dx - \int_{\tilde{x}(\theta)}^1 \frac{2F(x)}{2+\theta(n-1)}dx]$$

substitute $x^I(1) = \frac{1-(n-1)(1-\theta)x^U}{2+\theta(n-1)}$ and $F(1) = 1$ for when $v = 1$, then consolidate x^U and clean up:

$$\begin{aligned} x^U(\theta) = & \quad \frac{1}{A}[(1-\theta(n-1)[\frac{1-(n-1)(1-\theta)x^U(\theta)}{2+\theta(n-1)}]) \\ & -\int_0^{\tilde{x}(\theta)} F(x)dx - \int_{\tilde{x}(\theta)}^1 \frac{2F(x)}{2+\theta(n-1)}dx] \\ (A - \frac{\theta(1-\theta)(n-1)^2}{2+\theta(n-1)})x^U(\theta) = & \quad (1-[\frac{\theta(n-1)}{2+\theta(n-1)}]) - \int_0^{\tilde{x}(\theta)} F(x)dx - \int_{\tilde{x}(\theta)}^1 \frac{2F(x)}{2+\theta(n-1)}dx \\ (\frac{4+2\theta(n-1)+2(1-\theta)(n-1)}{2+\theta(n-1)})x^U(\theta) = & \quad ([\frac{2+\theta(n-1)-\theta(n-1)}{2+\theta(n-1)}]) - \int_0^{\tilde{x}(\theta)} F(x)dx - \int_{\tilde{x}(\theta)}^1 \frac{2F(x)}{2+\theta(n-1)}dx \\ x^U(\theta) = & \quad \frac{1}{2(n+1)}[2-(2+\theta(n-1))\int_0^{\tilde{x}(\theta)} F(x)dx - 2\int_{\tilde{x}(\theta)}^1 F(x)dx] \\ x^U(\theta) = & \quad \frac{1}{2(n+1)}[2-2\int_0^1 F(x)dx - \theta(n-1)\int_0^{\tilde{x}(\theta)} F(x)dx] \\ x^U(\theta) = & \quad \frac{1}{2(n+1)}[2\int_0^1 1-F(x)dx - \theta(n-1)\int_0^{\tilde{x}(\theta)} F(x)dx] \end{aligned}$$

Proof of Item (4)

Simply plug $\theta = 1$ into the formulas. Notice that $\tilde{x}(1) = 0$.

$$x^U(1) = \frac{\int_0^1 1 - F(v)dv + (n-1)\int_0^0 F(v)dv}{n+1} = \frac{\int_0^1 1 - F(v)dv}{n+1},$$

and

$$\bar{x}^I(1) = \frac{\int_0^1 1 - F(v)dv}{n+1}.$$

Therefore $x^U(1) = \bar{x}^I(1)$.

□

Proposition 2 (Proposition 2)**.**

1. *There exists $\theta' \in (0,1)$ at which $x^U(\theta)$ is minimized.*
2. *There exists $\theta^* \in (0,1)$ at which $\bar{X}(\theta)$ is maximized.*
3. *Total Donations are minimized at $\theta \in \{0,1\}$.*

Proof. **Proof of Item (1.)**

To proof the existence of a minimum for x^U, I will use Rolle's Theorem. To do this I need to show the endpoints are equal, and that the slope at $\theta = 0$ is negative.

Showing the endpoints are equal:

From Proposition 1 we know that $x^U = \frac{1}{2(n+1)}[2\int_0^1 1 - F(v)dv - \theta(n-1)\int_0^{(n-1)(1-\theta)x^U(\theta)} F(v)dv]$. Plug in $\theta = \{0,1\}$.

$$x^U(0) = \frac{\int_0^1 1 - F(v)dv}{(n+1)}$$

and

$$
\begin{aligned}
x^U(1) &= \tfrac{2\int_0^1 1-F(v)dv-(n-1)\int_0^0 F(x)dx}{2(n+1)} \\
&= \tfrac{\int_0^1 1-F(v)dv}{(n+1)}
\end{aligned}
$$

Thus, $x^U(0) = x^U(1)$.

Showing x^U initially decreasing:

Performing the differential we find $\frac{\partial x^U(\theta)}{\partial\theta} = \frac{(n-1)[\theta(n-1)F(\tilde{x}(\theta))x^U(\theta)-\int_0^{\tilde{x}(\theta)} F(v)dv]}{2(n+1)+(n-1)^2(1-\theta)\theta F(\tilde{x}(\theta))}$, where $\tilde{x}(\theta) = (n-1)(1-\theta)x^U(\theta)$.

Plug in $\theta = 0$ to get: $\frac{\partial x^U(0)}{\partial\theta} = \frac{(n-1)[-\int_0^{(n-1)x^U(0)} F(v)dv]}{2(n+1)}$, which is negative because $F(v)$ is a continuous distribution.

Therefore, applying Rolle's theorem, this implies there exists a $\theta' \in (0,1)$, at which $x^U(\theta')$ is a minimum.

Proof of Item (2.) This proof will also proceed by use of Rolle's Theorem.

Showing the endpoints are equal:

Recall that $\bar{X}(\theta) = (n)[\theta\bar{x}^I(\theta) - (1-\theta)x^U(\theta)]$. Therefore for $\bar{X}(0) = (n)[x^U(0)] = \frac{n[\int_0^1 1-F(v)dv]}{(n+1)}$.

$$
\begin{aligned}
\bar{X}(1) = &\quad n\bar{x}^I(1) \\
= &\quad n\tfrac{\int_{(n-1)(1-1)x^U(1)}^1 1-F(v)dv}{2+(n-1)} \\
= &\quad \tfrac{n[\int_0^1 1-F(v)dv]}{(n+1)}
\end{aligned}
$$

Therefore $\bar{X}(0) = \bar{X}(1)$.

Showing $\bar{X}$ is initially increasing:

Partial expected total donations to get

$$\frac{\partial \bar{X}(\theta)}{\partial \theta} = \underbrace{\bar{x}^I(\theta) - x^U(\theta)}_{(I)} + \underbrace{\frac{\partial x^U(\theta)}{\partial \theta}}_{(II)} + \underbrace{\theta(\frac{\partial \bar{x}^I(\theta)}{\partial \theta} - \frac{\partial x^U(\theta)}{\partial \theta})}_{(III)}$$

At $\theta = 0$, that simplifies to two terms (I) and (II). We know (I) from proof in Proposition 1, and we know (II) from proof in Proposition 2. Input to get:

$$\begin{aligned}
\frac{\partial \bar{X}(0)}{\partial \theta} &= \frac{\int_0^{\tilde{x}(\theta)} F(v)dv + \frac{\theta(n-1)}{2}\int_{\tilde{x}(\theta)}^1 F(v)dv}{2+\theta(n-1)} + \frac{(n-1)[\theta(n-1)F(\tilde{x}(\theta))x^U(\theta) - \int_0^{\tilde{x}(\theta)} F(v)dv]}{2(n+1)+(n-1)^2(1-\theta)\theta F(\tilde{x}(\theta))} \\
\frac{\partial \bar{X}(0)}{\partial \theta} &= \frac{\int_0^{(n-1)x^U(0)} F(v)dv}{(2)} - \frac{(n-1)[\int_0^{(n-1)x^U(0)} F(v)dv]}{2(n+1)} \\
&= \frac{1}{(n+1)}[\int_0^{(n-1)x^U(0)} F(v)dv] > 0
\end{aligned}$$

Therefore, expected total donations is initially increasing. Now we apply Rolle's Theorem, which means there must exist a $\theta^* \in (0,1)$ that maximizes expected total donations.

Proof of Item (3.)

I want to show that that total donations are minimized at the endpoints. Consider $\frac{n-1}{n}[\bar{X}(\theta) - \bar{X}(0)] > 0$. Applying multiple steps of algebra to verify the inequality becomes:

$$\begin{aligned}
0 <\ & \frac{n-1}{n}[\bar{X}(\theta) - \bar{X}(0)] \\
=\ & \frac{\theta(n-1)}{2+\theta(n-1)}\int_{\tilde{x}(\theta)}^1 1 - F(v)dv + \frac{(1-\theta)(n-1)}{n+1}\mu - \frac{(1-\theta)\theta(n-1)^2}{2(n+1)}\int_0^{\tilde{x}(\theta)} F(v)dv - \frac{n-1}{n+1}\mu \\
0 <\ & (\theta(n-1)) * [\frac{1}{2+\theta(n-1)}\int_{\tilde{x}(\theta)}^1 1 - F(v)dv - \frac{(1-\theta)(n-1)}{2(n+1)}\int_0^{\tilde{x}(\theta)} F(v)dv - \frac{\mu}{n+1}] \\
=\ & \frac{1}{2+\theta(n-1)}\int_{\tilde{x}(\theta)}^1 1 - F(v)dv - \frac{(n-1)}{2(n+1)}\int_0^{\tilde{x}(\theta)} F(v)dv - x^U(\theta) \\
=\ & \frac{1-(n+1)x^U(\theta) - \int_{\tilde{x}(\theta)}^1 F(v)dv}{2+\theta(n-1)} - \frac{(n-1)}{2(n+1)}\int_0^{\tilde{x}(\theta)} F(v)dv \\
=\ & (n+1)(1 - \mu + \frac{(n-1)\theta}{2}\int_0^{\tilde{x}(\theta)} F(v)dv - \int_{\tilde{x}(\theta)}^1 F(v)dv) - \frac{(2+\theta(n-1))(n-1)}{2}\int_0^{\tilde{x}(\theta)} F(v)dv.
\end{aligned}$$

Now, simplify $\frac{(n+1)(n-1)\theta - 2(n-1) - (n-1)^2\theta}{2}\int_0^{\tilde{x}}(\theta)F(v)dv = -(1-\theta)(n-1)\int_0^{\tilde{x}(\theta)} F(v)dv$, and substitute it in to get:

$$
\begin{aligned}
&= (n+1)(1-\mu-\int_{\tilde{x}(\theta)}^{1} F(v)dv) - (1-\theta)(n-1)\int_0^{\tilde{x}(\theta)} F(v)dv \\
&= (n+1)(1-\int_0^1 1-F(v)dv - \int_{\tilde{x}(\theta)}^{1} F(v)dv) - (1-\theta)(n-1)\int_0^{\tilde{x}(\theta)} F(v)dv \\
&= (n+1)(\int_0^1 F(v)dv - \int_{\tilde{x}(\theta)}^{1} F(v)dv) - (1-\theta)(n-1)\int_0^{\tilde{x}(\theta)} F(v)dv \\
&= (n+1)\int_0^{\tilde{x}(\theta)} F(v)dv - (1-\theta)(n-1)\int_0^{\tilde{x}(\theta)} F(v)dv \\
&= (n+1)-(1-\theta)(n-1) > 0
\end{aligned}
$$

And the last part is positive. Since $\bar{X}(1) = \bar{X}(0)$, this proves that both endpoints are the minimum total donations.

□

Lemma 1.

- $\Delta(\theta) > 0$, *for* $\theta \in [0,1]$, *with* $\Delta(0) > \Delta(1)$, *and* $\Delta'(1) > 0$.
- $\Delta(\theta)$ *is strictly decreasing in* K.

Proof. **Proof of Item (1.)**

The value of information is always positive, $\Delta(\theta) > 0$.

Since $x^I(v,\theta)$ is the argument that maximizes informed utility at each v, then we see for v such that $x^I(v,\theta) \neq x^U(\theta)$, gives:

$$\frac{1}{K}(v-x^I(v,\theta))*(x^I(v,\theta)+(n-1)x(v,\theta)) > \frac{1}{K}(v-x^U(\theta))(x^U(\theta)+(n-1)*x(v,\theta)).$$

Since the informed utility is greater for all values of v where individual informed donations different from uninformed donations, and the fact that F is a continuous distribution, the expected indirect utility is also always greater.

$$\begin{aligned}\frac{1}{K}\int_0^1 (v - x^I(v,\theta)) * (x^I(v,\theta) + & \quad (n-1)x(v,\theta))dF(v) > \\ & \frac{1}{K}\int_0^1 (v - x^U(\theta))(x^U(\theta) + (n-1) * x(v,\theta))dF(v)\end{aligned}$$

Therefore, $\Delta(\theta) > 0$ for all $\theta \in [0, 1]$.

To show $\Delta(0) > \Delta(1)$.

First, derive a slightly simpler formation of Δ under a single integration.

$$\begin{aligned}\Delta(\theta) = & \quad \frac{n-1}{K}\int_0^1 (x^U(\theta) - x^I(v,\theta)) * x(v,\theta)dF(v) \\ & +\frac{1}{K}\int_0^1 v(x^I(v,\theta) - x^U(\theta)) + x^U(\theta)^2 - x^I(v,\theta)^2 dF(v) \\ \Delta(\theta) = & \quad \frac{1}{K}\int_0^1 (x^I(v,\theta) - x^U(\theta))(v - (n-1) * x(v,\theta) - x^U(\theta) - x^I(v,\theta))dF(v)\end{aligned}$$

Then, $\Delta(0) > \Delta(1)$ becomes:

$$\begin{aligned}\int_0^1 (x^I(v,0) - \frac{\mu}{n+1})(v - n\frac{\mu}{n+1} & \quad -x^I(v,0))dF(v) > \\ & \int_0^1 (\frac{v}{n+1} - \frac{\mu}{n+1})(v - (n) * \frac{v}{n+1} - \frac{\mu}{n+1})dF(v).\end{aligned}$$

Simplifying we get:

$$\int_0^{\frac{(n-1)\mu}{(n+1)}} (-\frac{\mu}{(n+1)^2})(v + n(v-\mu))dF(v) + \int_{\frac{(n-1)\mu}{(n+1)}}^1 (\frac{v-\mu}{2})^2 dF(v) > \int_0^1 (\frac{v-\mu}{n+1})^2 dF(v).$$

Notice that the difference between the Δ is growing as n grows, therefore we assume $n = 2$.

Further notice,

$$(-\frac{\mu}{(n+1)^2})\int_0^{\frac{(n-1)\mu}{(n+1)}} ((n+1)v - n\mu)dF(v)$$

is positive, because $(n+1)v - n\mu$ is always negative over the range from 0 to $\frac{(n-1)\mu}{n+1}$. Therefore, we simplify the problem by dropping it and considering inequality without.

$$\Delta(0) - \Delta(1) > \frac{1}{4}\int_{\frac{(n-1)\mu}{(n+1)}}^{1} (v-\mu)^2 dF(v) - \frac{1}{9}\int_0^1 (v-\mu)^2 dF(v) > 0.$$

and reduce by algebra:

$$\begin{aligned}
0 <\ & 9\int_{\frac{(n-1)\mu}{(n+1)}}^{1} (v-\mu)^2 dF(v) - 4\int_0^1 (v-\mu)^2 dF(v) \\
0 <\ & 5\int_{\frac{(n-1)\mu}{(n+1)}}^{1} (v-\mu)^2 dF(v) - 4\int_0^{\frac{(n-1)\mu}{n+1}} (v-\mu)^2 dF(v) \\
0 <\ & 5\int_0^1 (v-\mu)^2 dF(v) - 5\int_0^{\frac{(n-1)\mu}{n+1}} (v-\mu)^2 dF(v) \\
& -4\int_0^{\frac{(n-1)\mu}{n+1}} (v-\mu)^2 dF(v).
\end{aligned}$$

$$\begin{aligned}
\text{Now substitute}\quad & \int_0^1 (v-\mu)^2 dF(v) = 2\int_0^{\mu} (v-\mu)^2 dF(v) \\
0 <\ & 5\int_0^1 (v-\mu)^2 dF(v) - \tfrac{9}{2}\int_0^1 (v-\mu)^2 dF(v), \\
0 <\ & \tfrac{1}{2}\int_0^1 (v-\mu)^2 dF(v),
\end{aligned}$$

which is positive. Therefore, $\Delta(0) > \Delta(1)$.

To show Δ increasing at $\theta = 1$.

Intuitively, the value of information is always positive. The slope of the value of information is:

$$\Delta'(\theta) = \ \tfrac{n-1}{K}\int_0^1 [x^U(\theta) - x^I(v,\theta)][\tfrac{\partial x(v,\theta)}{\partial\theta}]dF(v)$$

If we calculate the left derivative of $\frac{\partial x^I(v,\theta)}{\partial\theta}$ using the limit definition we get:

$$\begin{aligned}
\lim_{t\to 0^+} \tfrac{x^I(v,1-t)-x^I(v,1)}{-t} &= \tfrac{max\{0,\frac{v-t(n-1)x^U(1-t)}{2+(1-t)(n-1)}\}-\frac{v}{n+1}}{-t} \\
&= \lim_{t\to 0^+} \tfrac{\frac{-v}{n+1}\mathbb{1}_{v\leq t(n-1)x^U(1-t)}}{-t} \\
&\quad + \lim_{t\to 0^+} \tfrac{t(n-1)[v-(n+1)x^U(1-t)]}{-t(n+1)(2+(1-t)(n-1))}\mathbb{1}_{v\geq t(n-1)x^U(1-t)} \\
&= 0 + \tfrac{-(n-1)(v-(n+1)\frac{\mu}{n+1})}{(n+1)^2} \\
&= \tfrac{-(n-1)[v-\mu]}{(n+1)^2}.
\end{aligned}$$

which included in the definition of $\Delta'(\theta)$, at $\theta = 1$, becomes

$$\Delta'(1) = \frac{(n-1)^2}{(n+1)^3 K}\int_0^1 (v-\mu)^2 dF(v),$$

where $\Delta'(1) > 0$.

Proof of Item (2.) Notice, K is only in the denominator of $\Delta(\theta)$, therefore it is clearly decreasing in K. $\square$

Proposition 3. *For each cost c, the set of endogenous equilibria $\Theta(c)$ would be:*

for $c \geq \max_\theta \Delta(\theta)$**,** *fully uninformed is the only equilibrium in $\Theta(c)$.*

$$\bar{X}(0) = \frac{n}{n+1}[\int_0^1 1 - F(v)dv]$$

for $c < \min_\theta \Delta(\theta)$**,** *fully informed is the only equilibrium.*

$$\bar{X}(1) = \frac{n}{n+1}[\int_0^1 1 - F(v)dv]$$

for $\max_\theta \Delta(\theta) > c > \min_\theta \Delta(\theta)$**,** *there are partially-informed equilibria $\theta^c \in \Theta^c$, where $\theta^c \in$*

Θ^c *solves* $\Delta(\theta^c) = c$.

$$\bar{X}(\theta^c) = \tfrac{n\theta^c}{2+\theta^c(n-1)}[\int_{\tilde{x}(\theta^c)}^{1} 1-F(v)dv] + \tfrac{n(1-\theta^c)}{2(n+1)}[2\int_0^1 1-F(v)dv - \theta^c(n-1)\int_0^{\tilde{x}(\theta^c)} F(v)dv]$$

Proof. Therefore, if the cost of information c is greater than $\max_\theta \Delta(\theta)$, no one would prefer to become informed because the cost of information is always greater than the value of information, and the expected total donations would be the uninformed equilibrium, shown in proof for Proposition 2.

Likewise, if information is cheap, $c < \min_\theta \Delta(\theta)$, then all donors would prefer to become informed for all $\theta \in [0, 1)$ because information is always strictly valuable, see lemma 1. Thus, resulting in the fully informed equilibrium, with the same expected total donations as in fully uninformed, also shown in proof of Proposition 2.

Finally, if c is less than the max value of information and the minimum value of information, we will see partial equilibrium. Further, using fact that $\Delta(0) > \Delta(1)$ in Lemma 1, if $c > \Delta(1)$ then we will have partial equilibria that do not include fully informed, and vise versa if $\Delta(0) < c$. Thus, it is possible to choose c such that the only equilibria in $\Theta(c)$ are partial-equilibria. Finally, $\bar{X}(\theta^c)$ known from Proposition 1 and definition of $\bar{X}(\theta^c)$. □

Proposition 4. *For any cost* $c' \in [0, \min_\theta \Delta(\theta))$*, there exists a cost* $c'' > c'$ *such that* $\bar{X}(\theta^{c''}) > \bar{X}(1)$*, where* $\theta^{c''} \in [0, 1]$ *satisfies* $\Delta(\theta^{c''}) = c''$.

Proof. Notice, for any $c' \in [0, \min_\theta \Delta(\theta))$, the expected total donations in equilibrium would be $\bar{X}(1)$, as can be seen in Proposition 3.

To prove, choose $c' = 0$. Choose $c'' > \Delta(1)$. Thus, $\theta^{c''} \in [0, 1)$ that solves for $\Delta(\theta^{c''}) = c''$ results in $\bar{X}(\theta^{c''}) > \bar{X}(1)$ by Lemma 1, because $\bar{X}(1)$ is minimum total donations as shown in Proposition 2. □